

## Numerical Questions

1	Population – unit 3 .....	3
2	Environmental Chemistry .....	5
2.1	Total organic carbon.....	5
2.2	Molarity, molality, normality .....	5
2.3	Ppb, ppm, ug/m <sup>3</sup> .....	8
2.3.1	Converting ppm to mass per unit volume .....	8
2.3.2	Converting ppb to mass per unit volume .....	8
2.3.3	Questions.....	9
2.4	pE, pH, pOH.....	9
2.5	Heat of combustion .....	12
2.6	Retention factor .....	15
2.7	Transmittance/absorbance/concentration .....	15
3	Noise pollution – unit 6.....	18
4	Air pollution – unit 6.....	22
5	Water pollution – unit 6 .....	25
5.1	Dimensions of wastewater treatment plant .....	25
5.2	Amount of pollutant/BOD mixing into water .....	25
5.3	Biochemical Oxygen Demand.....	26
5.3.1	Important formulae .....	26
5.3.2	Questions.....	28
6	Radioactivity and half-life – unit 2, 5 and 6 .....	30
7	Solar energy – unit 5 .....	31
8	Nuclear energy – unit 5.....	34
9	Magnetohydrodynamic power – unit 5 .....	38
10	Wind energy – unit 5.....	39
11	Coal power – unit 5.....	41

## Table of contents

12	Tidal power – unit 5 .....	43
13	Dam – unit 5.....	44
14	Ocean waves – unit 5 .....	45
15	Energy consumption – unit 5 .....	46
16	Fuel cells – unit 5.....	47
17	Aquifers – unit 4 .....	48
18	Volcanoes – unit 4 .....	50
19	CFC number – unit 10.....	51
20	Probability (unit 9, p752-766).....	52
21	Statistics (unit 9, p752-766).....	53
21.1	Harmonic mean .....	53
21.2	Geometric mean .....	53
21.3	Percentiles, Percentile ranks, Medians, Quartiles .....	54
21.4	Standard error of the mean aka margin of error .....	55
21.5	Standard deviation.....	55
21.6	combined standard deviation and/or variance .....	56
21.7	Confidence interval .....	56
21.8	Regression .....	57
21.9	Correlation coefficient.....	58
21.10	Residual (unexplained) and overall variance .....	59
21.11	Chi sqr or a goodness of fit test.....	60
21.12	ANOVA and f-statistic.....	60
21.13	t-statistic .....	61
21.14	significance level.....	62
21.15	binomial distribution, mean and SD or variance.....	63
21.16	Skewness .....	64
21.17	Moments of distribution .....	64

## 1 Population – unit 3

Size of population growth (Q5, 203) <https://www.youtube.com/watch?v=-DDK1GWMuRA>

1. A country has population of 350 mn and an annual growth rate of population 2% at  $t=0$ . Thereafter the population stabilises at 1.4 bn. How long will it take for the population to reach 0.7 bn? (Q5, p551; UGC NET Jun 2019)

In a logistic growth model, instantaneous rate of change of population,  $dN/dt$  is given by  $\frac{dN}{dt} = rN(1 - \frac{N}{K}) = rN - \frac{rN^2}{K}$

Here,  $dN = 0.7 \text{ bn} - 350 \text{ mn} = 700 \text{ mn} - 350 \text{ mn} = 350 \text{ mn}$

$r = 0.02$

$N = 350 \text{ mn}$

$K = 1.4 \text{ bn} = 1400 \text{ mn}$

Substituting values, we get

$$\begin{aligned} \frac{350 \text{ mn}}{dt} &= 0.02 \times 350 \text{ mn} \left(1 - \frac{350 \text{ mn}}{1400 \text{ mn}}\right) \\ &= 0.02 \times 350 \text{ mn} \left(1 - \frac{1}{4}\right) = 0.02 \times 350 \text{ mn} \left(\frac{3}{4}\right) \end{aligned}$$

Solving this, we get  $dt = 200/3 = 66.66$  years.

2. In an ecosystem following logistic growth model, initial population was 900 with growth rate constant of 0.1. If the carrying capacity of the ecosystem is 1000, what is the instantaneous rate of change of population? (Q203, p571; UGC NET Jul 2016)

In a logistic growth model, instantaneous rate of change of population,  $dN/dt$  is given by

$$rN\{1-(N/K)\} = 0.1 \times 900 (1 - (900/1000)) = 9$$

3. The population ( $N$ ) of a fish in a pond follows the logistic equation  $\frac{dN}{dt} = 0.1N - 0.001N^2$ . What is the maximum sustainable yield.

In a logistic growth model, instantaneous rate of change of population,  $dN/dt$  is given by  $\frac{dN}{dt} = rN(1 - \frac{N}{K}) = rN - \frac{rN^2}{K}$

Often, this is simplified as  $aN - bN^2$ , where  $a = r$  and  $b = r/K$ .

According to the question, the value of  $a = r = 0.1$  and  $b = r/K = 0.001$ .

Maximum Sustainable Yield (MSY) is given by  $\frac{a}{2b} = \frac{0.1}{0.002} = 50$ .

## Numerical Questions

Also, as is evident from the above equation,  $MSY = \text{half the carrying capacity (K)} = K/2$

$$\rightarrow K = 2MSY.$$

So we can find MSY from K.

$$r/K = r/2MSY = 0.001 \rightarrow 0.1/2MSY = 0.001 \rightarrow MSY = 0.1/0.002 = 50.$$

<https://www.youtube.com/watch?v=-DDK1GWMuRA>

## 2 Environmental Chemistry

### 2.1 Total organic carbon

4. A lake has 2.5 mg/L of dissolved organic carbon. The dissolved organic matter concentration in the lake is approximately?

$$\text{Organic matter} = \text{Total organic carbon} \times 1.72 \rightarrow 2.5 \times 1.72 = 4.3 \text{ mg/L}$$

### 2.2 Molarity, molality, normality

5. Quantity of 5M HCl required for preparing 1000 ml of 0.1 M HCl solution is (Q201, p525; UGC NET Jun 2013)

0.1 M HCl solution has 0.1 moles in 1 litre. So, 1000 ml (i.e. 1 L), as is desired volume here, will have 0.1 moles as well.

5 M HCl has 5 moles in 1 litre. Since we require 0.1 moles from this 5M solution, we need to extract  $x$  litres from it such that it has 0.1 moles.

$$5 : 1 :: 0.1 : x \rightarrow 5x = 0.1 \rightarrow x = 0.1/5 = 0.02 \text{ litres or } 20 \text{ ml.}$$

6. Normality of 0.25 M phosphoric acid is? (Q173, p522; UGC NET Dec 2013)

In order to calculate normality, we need to know the formula of the substance. Phosphoric acid is  $\text{H}_3\text{PO}_4$ .

$$\text{Normality} = \text{molarity} \times \text{number of equivalents}$$

Here, the molarity is 0.25 M and number of equivalents is the number of hydrogen ions in an atom (or, more accurately, the number of moles of hydrogen ions in one mole of  $\text{H}_3\text{PO}_4$ ) is 3.

$$\text{So, Normality} = 0.25 \times 3 = 0.75\text{N } \langle \text{https://byjus.com/jee/normality/} \rangle$$

7. Number of cadmium ions present in 3n moles of cadmium nitrate is? (Q147, p520; UGC NET Jun 2014)

Formula for cadmium nitrate is  $\text{Cd}(\text{NO}_3)_2$

Normality = molarity  $\times$  number of equivalents (number of equivalents is the number of cadmium ions present in 1 molecule of cadmium nitrate. In this case, it is 1)

$$\text{Molarity} = 3 \text{ mol/L}$$

Since there are three moles of cadmium nitrate, there will be  $1 \times 3$  moles of cadmium ions i.e.  $3 \times 6.023 \times 10^{23} = 18.069 \times 10^{23}$ . This is in the option list, but is the wrong answer according to UGC. Strangely, the right answer is  $18.069 \times 10^{14}$ , I do not know how. I do not know what '3n moles' means – it should either be '3 moles' or 'normality 3'.

8. Calculate the approximate volume of water that must be added to 250 ml of 1.25 N solution to make it 0.50 N. (Q387, p540; JNU PhD 2007)

## Numerical Questions

### *Method 1*

$$N_1V_1 = N_2V_2$$

$$N_1 = 1.25 \text{ N}$$

$$V_1 = 250 \text{ ml or } 0.25 \text{ l}$$

$$N_2 = 0.50 \text{ N}$$

$$V_2 = ?$$

Using the formula above,

$$1.25 \times 0.25 = 0.50 \times V_2 \rightarrow V_2 = 625 \text{ ml}$$

### *Method 2*

While Method 1 was a simpler way to solve a problem, if you don't know the  $N_1V_1=N_2V_2$  formula, the question can still be solved using the unitary method.

The change in normality is being effected by changing the volume of water and not by changing the amount of the chemical dissolved (which would change the molarity) or adding another chemical (which would change the equivalent weight). In other words, the actual number of moles dissolved remains the same, but molarity changes because of the change in volume.

So,

Normality in solution 1 = (number of moles in solution 1/ volume in solution 1)  $\times$  eq. wt.

$$\rightarrow 1.25 = (\text{number of moles in solution 1} / 0.25 \text{ l}) \times \text{eq. wt.}$$

$$\rightarrow \text{number of moles in solution 1} = (1.25 \times 0.25) / \text{eqwt}$$

And,

Normality in solution 2 = (number of moles in solution 2/ volume in solution 2)  $\times$  eq. wt.

Let's say volume of solution 2, which the question asks, is  $x$ .

$$\rightarrow 0.50 = (\text{number of moles in solution 2} / x \text{ litre}) \times \text{eq. wt.}$$

$$\rightarrow \text{number of moles in solution 2} = (0.5x) / \text{eqwt}$$

Since the number of moles in solution 1 = number of moles in solution 2,

$$(1.25 \times 0.25) / \text{eqwt} = (0.5x) / \text{eqwt} \rightarrow x = 625 \text{ ml}$$

9. When 100 ml of 0.1 M of sulphuric acid is used to neutralise 0.01 N NaOH, how many ml of NaOH will be required? (Q469, p547; UGC NET Jun 2005)

## Numerical Questions

Let  $V_1$  be the volume of sulphuric acid = 100 ml or 0.1 litre

Let  $N_1$  be the normality of sulphuric acid = molarity  $\times$  eq. wt.

Since there are two hydrogen ions in sulphuric acid, eq. wt. is 2.

Thus,  $N_1 = 0.1 \times 2 = 0.2 \text{ N}$

Let  $V_2$  be the volume of NaOH required.

Let  $N_2$  be the normality of NaOH = 0.01 N

$N_1V_1 = N_2V_2$

$0.2 \times 0.1 = 0.01 V_2 \rightarrow V_2 = 2 \text{ l} = 2000 \text{ ml}$

10. Calculate mole fraction of water in a mixture of 12g water, 108g acetic acid and 92 g of ethyl alcohol (Q390, p541; JNU PhD 2007)

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often expressed in terms of  $\mu\text{g ml}^{-1}$  or ppm.

- (v) *Mole fraction*: Commonly used symbol for mole fraction is  $x$  and subscript used on the right hand side of  $x$  denotes the component. It is defined as:

Mole fraction of a component =

$$\frac{\text{Number of moles of the component}}{\text{Total number of moles of all the components}} \quad (2.4)$$

For example, in a binary mixture, if the number of moles of A and B are  $n_A$  and  $n_B$  respectively, the mole fraction of A will be

$$x_A = \frac{n_A}{n_A + n_B} \quad (2.5)$$

For a solution containing  $i$  number of components, we have:

$$x_i = \frac{n_i}{n_1 + n_2 + \dots + n_i} = \frac{n_i}{\sum n_i} \quad (2.6)$$

It can be shown that in a given solution sum of all the mole fractions is unity, i.e.

$$x_1 + x_2 + \dots + x_i = 1 \quad (2.7)$$

Mole fraction unit is very useful in relating some physical properties of solutions, say vapour pressure with the concentration of the solution and quite useful in describing the calculations involving gas mixtures.

(NCERT 2007: 37)

Molar mass of water = 18 g (calculated using amu)

Molar mass of acetic acid = 60 g

Molar mass of ethyl alcohol = 46 g

Using the above formula, mole fraction of water =

$$(12/18) \div \{(12/18) + (108/60) + (92/46)\} = 0.147$$

### 2.3 Ppb, ppm, ug/m<sup>3</sup>

) **Parts per million:** When a solute is present in **trace** quantities, it is convenient to express concentration in **parts per million (ppm)** and is defined as:

Parts per million =

$$\frac{\text{Number of parts of the component}}{\text{Total number of parts of all components of the solution}} \times 10^6 \quad (2.3)$$

As in the case of percentage, concentration in parts per million can also be expressed as mass to mass, volume to volume and mass to volume. A litre of sea water (which weighs 1030 g) contains about  $6 \times 10^{-3}$  g of dissolved oxygen (O<sub>2</sub>). Such a small concentration is also expressed as 5.8 g per 10<sup>6</sup> g (5.8 ppm) of sea water. The concentration of pollutants in water or atmosphere is often expressed in terms of  $\mu\text{g mL}^{-1}$  or ppm.

(NCERT 2007: 37)

#### 2.3.1 Converting ppm to mass per unit volume

For aqueous solutions, we need to find the volume of water that would have 10<sup>6</sup> mg (= 10<sup>3</sup> g = 1kg). The density of water is 10<sup>3</sup> kg/m<sup>3</sup>. If 1 m<sup>3</sup> has 10<sup>3</sup> kg, then 1 kg will be contained in 10<sup>-3</sup> m<sup>3</sup> (= 1 litre). So, we replace the denominator in  $\text{ppm} = 1 \text{ mg}/10^6 \text{ mg}$  with 1 litre and we get:

$$1 \text{ ppm} = 1 \text{ mg/L} = 10^{-3} \text{ g/L} = 1 \text{ g/m}^3$$

Even if we were to calculate the volume of water that would make up 10<sup>6</sup> g (instead of 10<sup>6</sup> mg), the answer would still have been the same as the ppm formula would have become  $1 \text{ g}/10^6 \text{ g}$ . Replacing the denominator with the appropriate volume would have resulted in  $1 \text{ g/m}^3$ , since 1 m<sup>3</sup> contains 10<sup>6</sup> g. This leads us to the same relationship as above.

For gaseous solutions, we need to find the volume of air that would have 10<sup>6</sup> mg (= 1kg). The density of air is 1 kg/m<sup>3</sup>. So, 1 m<sup>3</sup> contains 1 kg. We replace the denominator in  $\text{ppm} = 1 \text{ mg}/10^6 \text{ mg}$  with 1 m<sup>3</sup> and we get:

$$1 \text{ ppm} = 1 \text{ mg/m}^3 = 10^3 \mu\text{g/m}^3 \text{ (we can express this in the form of mg/L, but I guess litres are not used while talking of gases).}$$

#### 2.3.2 Converting ppb to mass per unit volume

For aqueous solutions, we need to find the volume of water that would have 10<sup>9</sup> mg (= 10<sup>6</sup> g = 10<sup>3</sup> kg). The density of water is 10<sup>3</sup> kg/m<sup>3</sup>. So, 1 m<sup>3</sup> has 10<sup>3</sup> kg = 10<sup>6</sup> g. Replacing the denominator in  $\text{ppb} = 1 \text{ mg}/10^9 \text{ mg}$  with 1 m<sup>3</sup> we get:

$$1 \text{ ppb} = 1 \text{ mg/m}^3 = 10^{-3} \text{ g/L} = 10^{-6} \text{ g/L} = 10^{-3} \text{ g/m}^3$$

For gaseous solutions, we need to find the volume of air that would have 10<sup>9</sup> mg (= 10<sup>6</sup> g = 10<sup>3</sup> kg). The density of air is 1 kg/m<sup>3</sup>. So, 10<sup>3</sup> m<sup>3</sup> contains 10<sup>3</sup> kg. We replace the denominator in  $\text{ppb} = 1 \text{ mg}/10^9 \text{ mg}$  with 10<sup>3</sup> m<sup>3</sup> and we get:

$$1 \text{ ppb} = 1 \text{ mg}/10^3 \text{ m}^3 = 10^{-3} \text{ mg/m}^3 = 1 \mu\text{g/m}^3$$



## Numerical Questions

<https://www.adelaide.edu.au/arcpoh/dperu/fluoride/ppm.html#:~:text=This%20is%20an%20abbreviation%20for,report%20means%20the%20same%20thing;>  
<https://www.youtube.com/watch?v=0ktAsc8y3hc>

### 2.3.3 Questions

11. An ozone analyser shows 100 ppb concentration at 1 atm and 25 °C. Convert the observed concentration of ozone in  $\mu\text{g}/\text{m}^3$ . (Q3, p506; UGC NET Jun 2019) Ans: 196.2.

Since  $1 \text{ ppb} = 10^{-3} \text{ g}/\text{m}^3$ ,  $100 \text{ ppb} = 10^{-1} \text{ g}/\text{m}^3 = 10^5 \mu\text{g}/\text{m}^3$ .

I do not understand how UGC's answer is  $196.2 \mu\text{g}/\text{m}^3$ .

12.  $3.0 \times 10^{18}$  molecules of  $\text{NO}_2$  are present in  $1.5 \times 10^{25}$  molecules of air at STP. Calculate the concentration of  $\text{NO}_2$  in ppb. (Q69, p512; UGC NET Aug 2016)

Using the formula above for ppm or ppb, I get

$$\{(3.0 \times 10^{18}) \div (1.5 \times 10^{25})\} \times 10^9 = 200 \text{ ppb}$$

13. Amount of 8-hydroxychloroquinolin (molecular weight 145.16 g) required for preparing 1000ml of 5 ppm solution is? (Q183, p523; UGC NET Sep 2013)

Since  $1 \text{ ppm} = 1 \text{ mg}/\text{L}$ , 5 ppm will be 5 mg/L. So, 1 litre requires 5 mg of solute.

14. A reference solution is to be prepared with 5 ppm parabenzo-pyrene (molecular weight 252 g). The quantity of parabenzo-pyrene required for preparing one litre 5 ppm solution is? (Q227, p527; UGC NET Dec 2012)

Since  $1 \text{ ppm} = 1 \text{ mg}/\text{L}$ , 5 ppm will be 5 mg/L. So, 1 litre requires 5 mg of solute.

15. The molecular weight of DDT is 354.5. the quantity of ddt required to prepare one litre of 10 ppm DDT solution is? (Q254, p529; UGC NET Jun 2012)

Since  $1 \text{ ppm} = 1 \text{ mg}/\text{L}$ , 10 ppm will be 10 mg/L. So, 1 litre requires 10 mg of solute.

### 2.4 pE, pH, pOH

(Q24, 107, 125, 156, 203, 223, 253, 263, 278, 330, 358, 372, 417)

16. Identify the most reducing water sample as inferred from pE values given below: 15, 12, 7, 5 (Q24, p508; UGC NET Jul 2018).

I do not know how to solve this question. The correct answer is 5, which probably means the lowest value in the options list is most reducing.

17. Based on the pE values for four water samples given below, the concentration of dissolved oxygen shall be highest in a) pE = -4.1, (b) pE = 13.9 (c) pE = 1.0 (d) pE = 7.0 (Q107, p516; UGC NET Dec 2015)

The correct answer is (b) pE = 13.9. Don't know why.

## Numerical Questions

Probably the higher the pE value, the higher the oxygen value and the less reducing it is. And the lower the pE value, the lower its oxygen value and the more reducing it is.

18. What is the pE value in an acid mine water sample having  $[\text{Fe}^{2+}] = 7.03 \times 10^{-3} \text{ M}$  and  $[\text{Fe}^{3+}] = 3.72 \times 10^{-4} \text{ M}$  (Q203, p525; UGC NET Jun 2013).  
<https://www.youtube.com/watch?v=pRCYIfxdawQ>

$$pE = pE^0 + (1/n)(\log_{10}Q) - (\text{the Nernst equation, Manahan 2017: 68})$$

pE = cell potential

$pE^0$  = standard cell potential

n = number of electrons involved in reaction (here, 1)

Q = concentration of reactants ÷ concentration of products

Here,  $pE^0 = 13.2$  (not given in the question, should have been; Manahan 2017: 102)

Substituting these values, we get

$$pE = 13.2 + \log(3.72 \times 10^{-4} / 7.03 \times 10^{-3}) = 13.2 + \log 0.0529 = 13.25$$

19. In pure water, the generation of each one of the ions  $\text{H}^+$  and  $\text{OH}^-$  at standard temperature and pressure? (Q330, p536; UGC NET Jun 2009)

At standard temperature (298K or 25 °C) and pressure (1 atm) (STP),

$[\text{H}^+][\text{OH}^-] = 10^{-14}$  (Masters and Ela 2014: 58), where the concentration of hydrogen and hydroxyl ions is equal i.e.  $[\text{H}^+] = [\text{OH}^-]$ .

So,  $[\text{H}^+][\text{OH}^-] = 10^{-14} \rightarrow [\text{H}^+]^2 = 10^{-14} \rightarrow [\text{H}^+] = 10^{-7}$ .  $10^{-7}$  is also  $[\text{OH}^-]$ .

20. pH of 1mM HCl is? (Q125, p518, UGC NET Jun 2015)

*Method 1*

$\text{pH} = -\log[\text{H}^+]$ , where  $[\text{H}^+]$  is the concentration of the Hydrogen ions in mol/litre (Masters and Ela 2014: 58).

Here, the concentration of the hydrogen ion is 1 millimoles/litre, or  $10^{-3}$  mol/litre.

So,  $\text{pH} = -\log(10^{-3}) \rightarrow \text{pH} = -(-3) = 3$

*Method 2 (easier method)*

$$[\text{H}^+] = 10^{-\text{pH}}$$

Here, the concentration of the hydrogen ion is 1 millimoles/litre, or  $10^{-3}$  mol/litre.

$[\text{H}^+] = 10^{-3}$ , so  $\text{pH} = 3$

21. pH of 0.01 M  $\text{HNO}_3$  is? (Q263, p530; UGC NET Jun 2012)

## Numerical Questions

### *Method 1*

$\text{pH} = -\log[\text{H}^+]$ , where  $[\text{H}^+]$  is the concentration of the Hydrogen ions in mol/litre (Masters and Ela 2014: 58).

Here, the concentration of the hydrogen ion is 0.01 moles/litre, or  $10^{-2}$  mol/litre.

$$\text{So, } \text{pH} = -\log(10^{-2}) \rightarrow \text{pH} = -(-2) = 2$$

### *Method 2 (easier method)*

$$[\text{H}^+] = 10^{-\text{pH}}$$

Here, the concentration of the hydrogen ion is 1 millimoles/litre, or  $10^{-2}$  mol/litre.

$$[\text{H}^+] = 10^{-2}, \text{ so } \text{pH} = 2$$

22. The pH of a solution containing hydrogen ions concentration of  $10^{-5}$  mol/L is? (Q417, p543; UGC NET Dec 2006)

### *Method 1*

$\text{pH} = -\log[\text{H}^+]$ , where  $[\text{H}^+]$  is the concentration of the Hydrogen ions in mol/litre (Masters and Ela 2014: 58).

Here, the concentration of the hydrogen ion is  $10^{-5}$  mol/L.

$$\text{So, } \text{pH} = -\log(10^{-5}) \rightarrow \text{pH} = -(-5) = 5$$

### *Method 2 (easier method)*

$$[\text{H}^+] = 10^{-\text{pH}}$$

Here, the concentration of the hydrogen ion is 1 millimoles/litre, or  $10^{-2}$  mol/litre.

$$[\text{H}^+] = 10^{-5}, \text{ so } \text{pH} = 5$$

23. pOH of 0.001 M HCl is? (Q223, p526; UGC NET Jun, Dec 2012, Dec 2011)

### *Method 1*

$\text{pH} = -\log[\text{H}^+]$ , where  $[\text{H}^+]$  is the concentration of the Hydrogen ions in mol/litre (Masters and Ela 2014: 58).

Here, the concentration of the hydrogen ion is 0.001 moles/litre, or  $10^{-3}$  mol/litre.

$$\text{So, } \text{pH} = -\log(10^{-3}) \rightarrow \text{pH} = -(-3) = 3$$

Subtract 3 from 14 (which is the maximum value on the pH scale), which will yield the pOH value i.e. 11 (=14 - 3)

### *Method 2*

## Numerical Questions

At standard temperature and pressure,  $[H^+][OH^-] = 10^{-14}$  (Masters and Ela 2014: 58). Here, the concentration of the hydrogen ion is 0.001 moles/litre, or  $10^{-3}$  mol/litre.

$$\text{So, } [10^{-3}][OH^-] = 10^{-14} = 10^{-11}$$

$pOH = -\log[OH^-]$ , where  $[OH^-]$  is the concentration of hydroxyl ions.

$$\text{So, } pOH = -\log[10^{-11}] = -(-11) = 11$$

24. A 50 ml solution of  $pH = 1$  is mixed with 50 ml solution of  $pH = 2$ . The  $pH$  of the mixture will be? (Q156, p521; UGC NET Jun 2014)

$$[H^+] \text{ for solution 1 w/ } pH = 1 = 10^{-1}; [H^+] \text{ for solution 2 w/ } pH = 2 = 10^{-2}$$

$$M_1V_1 + M_2V_2 = M_3V_3 \rightarrow 50 \times 0.1 + 50 \times 0.01 = 100 M_3 \rightarrow M_3 = 0.055$$

$$pH = -\log[H^+] = -\log 0.055 = -(-1.259) = 1.259$$

<https://www.youtube.com/watch?v=SCj2Gx400MU>

25. If  $pH$  of solution is 6.5, the concentration of  $[H^+]$  in mol/L will be? (Q372, p539; JNU PhD exam 2008)

$$[H^+] = 10^{-pH} = 10^{-6.5}$$

When faced with an exponent in decimals, just add the nearest higher integer. In this case, this becomes:

$$[H^+] = 10^{-6.5 + 7 - 7} = 10^{0.5 - 7} = 10^{0.5} \times 10^{-7} = 10^{1/2} \times 10^{-7} = 3.16 \times 10^{-7}$$

26. What is the  $[OH^-]$  ion concentration of HCL whose  $pH$  is 3? (Q185, p523; UGC NET Sep 2013)

$$\text{If the } pH \text{ is 3, then } pOH \text{ is } 14 - 3 = 11. \text{ So, } [OH^-] = 10^{-pOH} = 10^{-11}$$

At STP, hydrogen ion concentration in a solution is  $5 \times 10^{-4}$  mol/l. What is the concentration of the hydroxyl ions?

$$[H^+][OH^-] = 10^{-14} \rightarrow [OH^-] = (5 \times 10^{-4})/10^{-14} = 2 \times 10^{-11}$$

### 2.5 Heat of combustion

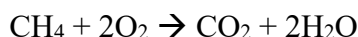
(Q94, 124, 394 <https://www.youtube.com/watch?v=at314m4QZgI>)

27. Given the standard enthalpies for methane, carbon dioxide and water vapour as -75 kJ/mol, 394 kJ/mol and 240 kJ/mol respectively, what is the net heat of combustion for methane? (Q94, p515; UGC NET Jul 2016)

Heat of combustion is calculated by subtracting the standard enthalpy of the reactants from the products side of a chemical reaction (Masters and Ela 2014: 53).

The reaction here is

## Numerical Questions



So, heat of combustion of  $\text{CH}_4$  is

Standard enthalpy ( $\text{CO}_2 + 2\text{H}_2\text{O}$ ) – standard enthalpy ( $\text{CH}_4 + 2\text{O}_2$ )

The standard enthalpy of  $\text{O}_2$ ,  $\text{N}_2$ ,  $\text{H}_2$  is 0 J at standard temperature and pressure (need to memorise, Masters and Ela 2014: 54).

So, heat of combustion for  $\text{CH}_4$  is

$$(-394 + 2(-240)) - (75 + 2(0)) = -799 \text{ kJ/mol}$$

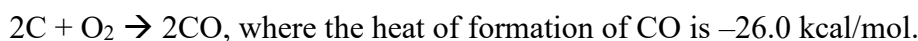
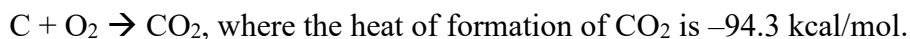
28. Burning of 1 mol of  $\text{CH}_4$  liberates 890 kJ of energy and 1 mol of  $\text{CO}_2$  has 12g carbon, the higher heating value of methane is? (Q124, p518; UGC NET Jun 2015)

There seems to be an error in the question, since all the options are not in kJ/mol but in grammes C/megajoule. The questioner probably intended to ask the amount of carbon produced per MJ of energy.

Since 12 g carbon is being produced per 890 kJ (=0.89 MJ) energy, the answer is  $(12/0.89) \text{ gC/MJ} = 13.483 \text{ gC/MJ}$ .

29. On combustion, carbon forms two oxides, CO and  $\text{CO}_2$ . Heat of formation of  $\text{CO}_2$  is  $-94.3 \text{ kcal}$  and that of CO is  $-26.0 \text{ kcal}$ . Heat of combustion of carbon is? (Q394, p541; JNU PhD 2007)

The reactions referred to in the question are:



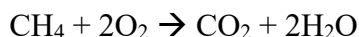
This question does not require any calculation, but a fairly simple concept.

Heat of combustion concept assumes that there is not dearth of oxygen in the atmosphere. In case of no shortage of oxygen, C forms  $\text{CO}_2$ . So, the heat of combustion for carbon is  $-94.3 \text{ kcal/mol}$ .

<https://www.youtube.com/watch?v=SCV0Tx83BDo>

30. If the energy released during combustion of methane is 900 kJ/mol, its carbon intensity is? (Q134, p669; UGC NET Jun 2014, Sep 2013)

Carbon intensity is the amount of carbon released per unit energy. The combustion of methane is given by



So, one mole of  $\text{CH}_4$  produces one mole  $\text{CO}_2$ . And each mole of  $\text{CO}_2$  (44 g) contains 12 g carbon. According to the question, each mole of methane gives 900 kJ.

So, carbon intensity is

## Numerical Questions

(amount of carbon released by 1 mole methane) / (amount of energy released by 1 mole methane) =  $12 / 900 = 0.0133 \text{ gC/kJ}$  or  $13.3 \text{ gC/MJ}$ .

31. When biomass is converted to CO and H<sub>2</sub>O, the energy made available is ~450 kJ per mole of carbon per unit of reduction level. What would be the heat of combustion per gram of methane? (Q96, p688; UGC NET Jul 2016)

Reaction in question is  $2\text{CH}_4 + 3\text{O}_2 \rightarrow 2\text{CO} + 4\text{H}_2\text{O}$

The reaction can be rewritten as  $\text{CH}_4 + 1.5\text{O}_2 \rightarrow \text{CO} + 2\text{H}_2\text{O}$

Now, 450 kJ is released with each mole of carbon. It takes one mole (16 g) of CH<sub>4</sub> of carbon to produce 1 mole of carbon. So, the energy released per gram of methane is

$450 \div 16 = 28.125 \text{ kJ}$  (I don't know why the answer is just double the amount i.e. 56.25 kJ).

32. The heats of combustion of carbon and carbon monoxide are  $-393.5$  and  $-283.5$  kJ/mol respectively. The heat of formation (in kJ) of carbon monoxide per mole is?

<https://www.youtube.com/watch?v=at3l4m4QZgI>

There are three reactions referred to here:

$\text{C} + \text{O}_2 \rightarrow \text{CO}_2$ , where the heat of combustion is  $-393.5 \text{ kJ/mol}$

$2\text{CO} + \text{O}_2 \rightarrow 2\text{CO}_2$ , where the heat of combustion is  $-283.5 \text{ kJ/mol}$

$2\text{C} + \text{O}_2 \rightarrow 2\text{CO}$ , where we need to find the heat of combustion.

The energy production in first reaction can be mathematically expressed as

Standard enthalpy (CO<sub>2</sub>) – (standard enthalpy (C) + standard enthalpy (O<sub>2</sub>)) =  $-393.5 \text{ kJ/mol}$

Since standard enthalpy (O<sub>2</sub>) = 0 kJ,

**→ standard enthalpy (CO<sub>2</sub>) – standard enthalpy (C) =  $-393.5 \text{ kJ/mol}$  (eq 1)**

The energy production in second reaction can be expressed as

$2 \times \text{standard enthalpy (CO}_2) - \{2 \times \text{standard enthalpy (CO)} + \text{standard enthalpy (O}_2)\} = -283.5 \text{ kJ/mol}$

Since there are two moles of CO being combusted,

$2 \times \text{standard enthalpy (CO}_2) - 2 \times \text{standard enthalpy (CO)} = 2 \times -283.5 \text{ kJ}$

Dividing by 2 throughout, we get

**→ standard enthalpy (CO<sub>2</sub>) – standard enthalpy (CO) =  $-283.5 \text{ kJ}$  (eq 2)**

To make the equation simpler from next time on, the second reaction can be rewritten as  $\text{CO} + \frac{1}{2}\text{O}_2 \rightarrow \text{CO}_2$ . It may look chemically inaccurate, but makes sense when we

## Numerical Questions

are just talking in terms of energy produced/consumed per mole. Also, it's much more simple to calculate.

We can rewrite the third reaction similarly as  $C + \frac{1}{2}O_2 \rightarrow CO$

The heat of formation for CO in the third reaction is

$$\text{standard enthalpy (CO)} - \{\text{standard enthalpy(C)} + \text{standard enthalpy (O}_2)\} = ?$$

$$\text{standard enthalpy (CO)} - \text{standard enthalpy(C)} = ?$$

subtracting Eq (2) from Eq (1), we get

$$\{\text{standard enthalpy (CO}_2) - \text{standard enthalpy (C)}\} - \{\text{standard enthalpy (CO}_2) - \text{standard enthalpy (CO)}\} =$$

$$\text{standard enthalpy (CO}_2) - \text{standard enthalpy (C)} - \text{standard enthalpy (CO}_2) + \text{standard enthalpy (CO)} =$$

$$\text{standard enthalpy (CO)} - \text{standard enthalpy (C)} = -393.5 - (-283.5) = -110 \text{ kJ/mol}$$

## 2.6 Retention factor

33. In an exam, four students working independently on a chromatographic analysis reported four retention factor ( $R_f$ ) values. R reported 0, S reported 0.5, T reported 1.3 and U reported 1. Which value is definitely incorrect?

Ans: T.  $R_f$  values can never be greater than 1.

34. In a gas chromatography experiment, the retention factor values for A and B were 0.5 and 0.125 respectively. If the distance travelled by solvent front is 12 cms, the distances travelled by A and B are:

$$R_f = \text{migration distance of substance} \div \text{migration distance of solvent front}$$

$$\rightarrow \text{migration distance of substance} = R_f \times \text{migration distance of solvent front}$$

$$R_f \text{ for A} = 12 \times 0.5 = 6$$

$$R_f \text{ for B} = 12 \times 0.125 = 1.5 \text{ } <https://www.youtube.com/watch?v=xqEmdXyHW-A>$$

## 2.7 Transmittance/absorbance/concentration

Q, 76, 190, 210, 226, 252, 153

35. Three coloured sheets of equal thickness are placed in a light beam. Each sheet absorbs 20% of the light incident upon it. What is the intensity of light transmitted after the third phase? (Q21, p508; UGC NET Jul 2018)

First sheet absorbs 20% (or 0.2) of the original intensity of light ( $x$ ). So, light intensity after first beam will be  $x - 0.2x = 0.8x$  i.e. 80% of the original light intensity.

Light intensity after second layer will be 80% of  $0.8x = 0.64x$

## Numerical Questions

Light intensity after third layer will be 80% of  $0.64x = 0.512x$  or 51.2% of the original intensity.

36. A solution of chemical A having 0.14 mol/L concentration has absorbance of 0.42. another solution of A under the same conditions has absorbance of 0.36. What is the concentration of the latter? (Q28, p508; UGC NET Nov 2017)

<https://www.youtube.com/watch?v=iP0ibYmt5TM>

This problem has to be solved with simple unitary method.

Let  $x$  be the concentration of the latter solution. The problem can be express as:

$$0.42 : 0.14 :: 0.36 : x \rightarrow x = (0.14 \times 0.36) \div 0.42 = 0.12 \text{ mol/L}$$

37. In a spectrophotometric cell of 2.0 cm path length, the solution of a substance shows the absorbance value of 1.0. If the molar absorptivity of the compound is  $2 \times 10^4$  L/mol-cm, calculate the concentration of the substance in solution. What is the concentration of the substance in that solution?

<https://www.youtube.com/watch?v=sKYEpN0g8dQ>

Absorbance = molar extinction coefficient  $\times$  concentration  $\times$  length of cuvette

$$1 = 2 \times 10^4 \times \text{concentration} \times 2 \rightarrow \text{concentration} = 25 \times 10^{-6} \text{ mol/L}$$

38. Molar extinction coefficient of malondialdehyde is  $0.155 \text{ Litre millimol}^{-1} \text{ cm}^{-1}$ . The concentration of malondialdehyde in a solution having an absorbance of 0.31 is?

<https://www.youtube.com/watch?v=BVG2OWqEJqw>

Absorbance = molar extinction coefficient  $\times$  concentration  $\times$  length of cuvette

Since length is not given here, we shall just knock it out of the formula, essentially assuming it to be just 1 cm.

$$\text{So, concentration} = 0.31/0.155 = 2 \text{ mM}$$

39. A textile industry effluent containing  $1 \times 10^{-6} \text{ mol L}^{-1}$  of an organic dye has 0.6 absorbance in a cell of 1.0 cm path length. The molar extinction coefficient of the dye is? <https://www.youtube.com/watch?v=eH7VRAUOdV4>

Absorbance = molar extinction coefficient  $\times$  concentration  $\times$  length of cuvette

molar extinction coefficient =  $0.6 \times 10^{-6} \text{ L/mol-cm}$ . (all the options given by UGC were wrong).

40. A student prepared a calibration curve by measuring the absorbance of five standard solutions of a compound at 300 nm. A cuvette with a path length of 5.0 cm was used in the spectrophotometer. The slope of the curve was  $300.0 \text{ L mol}^{-1}$ . The molar absorptivity of the compound is? <https://www.youtube.com/watch?v=6WyGCHcPFIU>

This question cannot be solved by the *Absorbance = molar extinction coefficient  $\times$  concentration  $\times$  length of cuvette* formula because it gives the slope of molar absorptivity, and is asking you for the molar absorptivity.



## Numerical Questions

Slope of molar absorptivity = molar absorptivity  $\times$  length

$$300 = \text{molar absorptivity} \times 5 \rightarrow \text{molar absorptivity} = 60 \text{ L/mol-cm}$$

Thermodynamics – Q169; Frequency – Q323; Risk of contamination – (Q65, 161); Instantaneous rate of oxidation - Q7; Spectrometry/molar absorptivity Q9, 104; Molar mass (Q20; ~~Transmission (Q21)~~; Titration (Q32, 39); A/F – 127; %w/w, v/v – 132; Stoichiometry – 139; Distribution coefficient – 193; TSS – 189; Hardness of water – 211; Redox potential – 292; kPa – 213; solubility – 279; Gibbs energy – 301; K<sub>sp</sub> – 302; residence time/lifespan (Q30; Henry's Law Constant (Q33, 95)

### 3 Noise pollution – unit 6

**Convert dB to Pa** (Q 174, 116) – solutions in Singh (2019: 303)... what if dB is 30, 50?

dB	Pa
0	$2 \times 10^{-5}$
20	$2 \times 10^{-4}$
40	$2 \times 10^{-3}$
60	$2 \times 10^{-2}$
80	0.2
100	2
120	20

41. Noise level of 80 dB corresponds to sound pressure level of (in Pa)? – 0.2 Pa or  $2 \times 10^5 \mu\text{Pa}$
42. Sound of 0 dB corresponds to how many Pa? –  $20 \mu\text{Pa}$

**Reduction in decibel (dB) level wrt distance** (Q141) – solutions in Singh (2019: 304)

43. A point source of sound produces a noise of 70 dB at a distance 20 m from it. What will be the noise level at 80 m from it? (Q141, p692)

For every doubling of distance, there is a reduction of 6 dB.

$$\text{So, } L_2 - L_1 = -6 \text{ dB} \rightarrow L_2 = -6 + L_1 = -6 + 70 = 64 \text{ dB}$$

So, 64 dB is the sound pressure level at 40 m.

$$\text{Applying the equation again, } L_2 = L_1 - 6 = 64 - 6 = 58 \text{ dB}$$

<https://www.youtube.com/watch?v=FrKu1gFmXZk>

**Value of background noise level ( $L_{90}$ )** (131) – solutions in Singh (2019: 307)

44. Given the following data of sound pressure levels (in dB) measured at equal intervals: 60, 62, 63, 59, 60, 80, 58, 59, 65, 57, 62. The value of the background noise level  $L_{90}$  is? (Q131, p691)

$L_{90}$  is the highest-minimum sound level that is present 90% of the time. In other words, it is the highest value that is exceeded 90% of the time. In this case, we will list the observations in increasing order:

57  
58  
59  
59  
60  
60  
62  
62

Numerical Questions

- 63
- 65
- 80

we find that the value 58 dB is the answer.

**Results of >2 sound levels at the same level (Q104, 80, 2) – solution on p305.**

45. On take-off an aeroplane generates noise level of 120 dB. If 5 such aeroplanes take off simultaneously what will be the noise level?

$$L_{s \text{ total}} = L_s + 10\text{Log}(n)$$

Where  $L_{\text{total}}$  = total sound level,  $L_s$  = sound produced by source,  $n$  = number of sources

$$L_{\text{total}} = 120 + 10\log(5) \rightarrow 120 + 6.98 = 126.98$$

Common Logarithm to a Number ( $\log_{10} x$ )	Log Value
Log 1	0
Log 2	0.3010
Log 3	0.4771
Log 4	0.6020
Log 5	0.6989
Log 6	0.7781
Log 7	0.8450
Log 8	0.9030
Log 9	0.9542
Log 10	1

<https://www.youtube.com/watch?v=QCLrH-h0ziE>

log values: <https://byjus.com/maths/value-of-log-1-to-10/>

**Results of >2 sound levels at different levels**

46. The resultant of two noise levels of 50 dB and 55 dB is?

Difference between two noise levels (dB)	Add this correction to the higher noise level (dB)
0	3
1	2.5
2	2.1
3	1.8
4	1.5
5	1.2
6	1.0
7	0.8
8	0.6
9	0.5

So, to calculate the resultant of 50 dB and 55 dB, add 1.2 to 55, which is 56.2 dB.

<https://www.youtube.com/watch?v=VXaTp1lp5Zs>

**Convert decibels (dB) to sound intensity (W/m<sup>2</sup>) (Q42, P682)**

47. Noise level of 70 dB corresponds to sound intensity of:

Sound in dB	Intensity in W/m <sup>2</sup>
0	10 <sup>-12</sup>
10	10 <sup>-11</sup>
20	10 <sup>-10</sup>
30	10 <sup>-9</sup>
40	10 <sup>-8</sup>
50	10 <sup>-7</sup>
60	10 <sup>-6</sup>
70	10 <sup>-5</sup>
80	10 <sup>-4</sup>
90	10 <sup>-3</sup>
100	10 <sup>-2</sup>
110	10 <sup>-1</sup>
120	10 <sup>0</sup> = 1
130	10 <sup>1</sup>
140	10 <sup>2</sup>
150	10 <sup>3</sup>
160	10 <sup>4</sup>

Instead of remembering the above table, the simple trick is that

1. Divide the value in decibels (dB) by 10.
2. Then raise 10 to that value.
3. Multiply 10<sup>-12</sup> to that value.

So, dividing 70 dB by 10, we get 7.

Raising 10 to that value, we get 10<sup>7</sup>.

Multiplying 10<sup>-12</sup> by 10<sup>7</sup>, we get 10<sup>-5</sup> W/m<sup>2</sup>.

Solution from Singh (2019: 303) and

<https://www.youtube.com/watch?v=1jPzqI9xXyw>

**What percentage of decibels (Q246, p701; Q328)**

48. If sound pressure level of 90 dB for 8 hrs represents 100% dose of noise, noise dose of 25% implies (UGC NET Dec 2010, Jun 2005; Q246, 328):

90dB for 8 hrs has been considered 100% by ISO and is not a hypothetical figure. Besides, an increase in 3dB is considered a doubling of sound pressure. So, 93dB for 8 hrs would mean 200% dose of noise (or a 100% increase).

## Numerical Questions

So, 87 dB for 8 hrs would be 50% of dose of noise. 84 dB for 8 hrs *or* 87 dB for 4 hrs or 90 dB for 2 hrs or 93 dB for 1 hr will be 25%. From  
<https://www.youtube.com/watch?v=hnXXNuFwL8c>  
<https://www.youtube.com/watch?v=q9yLRllycINE>.

*See Masters and Ela (2014: 410, 512, 507). See Smith (2005) for noise.*

## 4 Air pollution – unit 6

**Volume of hydrocarbon emissions**

49. In a city, 0.5 mn cars approximately 20 km on a given day. The average hydrocarbon emission rate is 2 g/car-km. Assuming that the average molecular weight of all emitted HCs is equal to that of CH<sub>4</sub>, how much volume of hydrocarbon is released in the city each day? (Q46, p683; UGC NET Nov 2017)

If 2 g HCs are released in one kilometre, each car releases 40 g HC in 20 kilometre. Since there are 0.5 mn cars, total mass of HC emissions is 20 mn g per day.

Molar mass of CH<sub>4</sub> = 16 g.

Number of moles of HCs released in a day = 20 mn ÷ 16 = 5/4 mn = 1.25 mn moles

1 mole of gas a STP occupies 22.4 × 10<sup>-3</sup> m<sup>3</sup>, so 1.25 mn moles will occupy

1.25 × 10<sup>6</sup> × 22.4 × 10<sup>-3</sup> = 28 × 10<sup>3</sup> m<sup>3</sup> = 2.8 × 10<sup>4</sup> m<sup>3</sup>. (Masters and Ela 2014: 469).

**Electrostatic precipitator** (Q 181, 185, 79) see Master and Ela 2014: 435

**EXAMPLE 9** Electrostatic Precipitator Area

An electrostatic precipitator with 6,000 m<sup>2</sup> of collector plate area is 97 percent efficient in treating 200 m<sup>3</sup>/s of flue gas from a 200-MW power plant. How large would the plate area have to be to increase the efficiency to 98 percent and to 99 percent?

**Solution** Rearranging (36) to solve for the drift velocity,  $w$ , gives

$$w = -\frac{Q}{A} \ln(1 - \eta) = -\frac{200 \text{ m}^3/\text{s}}{6,000 \text{ m}^2} \ln(1 - 0.97) = 0.117 \text{ m/s}$$

To achieve 98 percent efficiency, the area required would be

$$A_{98} = -\frac{Q}{w} \ln(1 - \eta) = -\frac{200 \text{ m}^3/\text{s}}{0.117 \text{ m/s}} \ln(1 - 0.98) = 6,690 \text{ m}^2$$

To achieve 99 percent, the area required would be

$$A_{99} = -\frac{200 \text{ m}^3/\text{s}}{0.117 \text{ m/s}} \ln(1 - 0.99) = 7,880 \text{ m}^2$$

That is about the area of two football fields! As these calculations suggest, the additional collector area required to achieve incremental improvements in collection efficiency goes up rapidly. To increase from 97 to 98 percent required 690 m<sup>2</sup> of added area, whereas the next 1 percent increment requires 1,190 m<sup>2</sup>.

(From Masters and Ela 2014: 435). The solution involves taking natural logs, and I do not know how that is possible without a calculator or a log table, which is how it will be in the exam.

**Stack of pollutants** (Q51, p683) <https://www.youtube.com/watch?v=37QYWZumuTM>  
<https://www.youtube.com/watch?v=ueVC-R0LMiY> (Masters and Ela 2014: 458)

The ground-level concentration of pollution directly downwind of the stack is of interest since pollution will be highest along that axis. With  $y = 0$ , (45) simplifies to

$$C(x, 0) = \frac{Q}{\pi u_H \sigma_y \sigma_z} \exp\left(\frac{-H^2}{2\sigma_z^2}\right) \quad (49)$$

The following example illustrates the use of the Gaussian plume equation.

The problem involves natural logarithms and exponents based on  $e$ . No point trying to solve this question in the exam because one does not have either log tables or calculators.

But there are some important points to remember:

Concentration of a pollutant ( $C$ )  $\propto H^{-2}$ , where  $H$  is the stack height (Q180, p696; UGC NET Dec 2013 and Q194, p697; UGC NET Jun 2013, Jun 2011)

The plume size  $\Delta h$  depends on the inside radius of the stack ( $r$ ) as  $\Delta h \propto r^2$  (Q210, p699; UGC NET Dec 2012)

Plume rise  $\Delta h$  depends on the stack velocity  $V_s$  as  $\Delta h \propto V_s^{-1}$  (Q245, p701; UGC NET Dec 2010)

Plume rise  $\Delta h$  depends on the wind speed  $U$  as  $\Delta h \propto U^{-1}$  (Q277, p704; UGC NET Jun 2008)

**Box model problems** (Masters and Ela 2014: 467).

50. The box model for an airshed over a city has the following parameter values: Length of the airshed ( $L$ ) = 24 km, Average wind speed ( $u$ ) = 4 mps. If the initial concentration of a pollutant over the city is zero, estimate the time at which the concentration of the pollutant reaches ~63% of its final value (Q81, p687; UGC NET Aug 2016)?

The time at which the pollutant reaches 63% of its final value is obtained by:

$$L \div u = 24000 \text{ m} / 4 \text{ mps} = 6000 \text{ seconds} = (6000/3600) \text{ hrs} = 1.66 \text{ hrs}$$

$$= 1' 39.6'' \approx 1' 40''$$

Additionally, the time at which the pollutant reaches 86% of its final value is given by  $2L/U = 12000$  seconds = 3.33 hours = 3' 19.8''  $\approx$  3' 20''.

The time at which the pollutant reaches 95% of its final value is given by  $3L/U = 18000$  seconds = 5' 0''. <https://www.youtube.com/watch?v=KM2LOVgGqCM>

Concentration of pollutants (C) varies with mixing height (H) as  $C \propto 1/\sqrt{H}$  (Q221, p699; UGC NET Jun 2012 and Q290, p705; UGC NET Dec 2007)

**Settling velocity of a particle**

Settling velocity of a particle  $\propto$  particle's diameter<sup>2</sup> (Q187, p696 and Q298, p706; UGC NET Sep 2013)

**Concentration of substances** – (Q19, p753; 104, p762) (see Masters and Ela 2014: 469-470)

**EXAMPLE 17 Evening Rush Hour Traffic**

Suppose within a square city, 15 km on a side, there are 200,000 cars on the road, each being driven 30 km between 4 PM and 6 PM, and each emitting 3 g/km of CO. It is a clear winter evening with a radiation inversion that restricts mixing to 20 m. The wind is bringing clean air at a steady rate of 1.0 m/s along an edge of the city. Use a box model to estimate the CO concentration at 6 PM if there was no CO in the air at 4 PM, and the only source of CO is cars. Assume that CO is conservative and that there is complete and instantaneous mixing in the box.

**Solution** The emissions per m<sup>2</sup>,  $q_s$ , would be

$$q_s = \frac{200,000 \text{ cars} \times 30 \text{ km/car} \times 3 \text{ g/km}}{(15 \times 10^3 \text{ m})^2 \times 3,600 \text{ s/hr} \times 2 \text{ hr}} = 1.1 \times 10^{-5} \text{ g/s-m}^2$$

Using (61), the concentration after 2 hours (7,200 s) would be

$$C(t) = \frac{q_s L}{uH} (1 - e^{-ut/L})$$

$$C(2 \text{ hr}) = \frac{1.1 \times 10^{-5} \text{ g/s-m}^2 \times 15 \times 10^3 \text{ m}}{1.0 \text{ m/s} \times 20 \text{ m}} \left[ 1 - \exp\left(\frac{-1.0 \text{ m/s} \times 7,200 \text{ s}}{15,000 \text{ m}}\right) \right]$$

$$= 3.2 \times 10^{-3} \text{ g/m}^3 = 3.2 \text{ mg/m}^3$$

which is considerably below both the 1-hr NAAQS for CO of 40 mg/m<sup>3</sup> and the eight-hour standard of 10 mg/m<sup>3</sup>. Any CO that was already in the air at 4 PM would, of course, increase this estimate. The time constant,  $L/u = 15,000 \text{ m}/(1 \text{ m/s}) = 15,000 \text{ s} = 4.2 \text{ hours}$ , suggests that in these 2 hours, the concentration is well below what it would become if these conditions were to continue.



## 5 Water pollution – unit 6

### 5.1 Dimensions of wastewater treatment plant

51. Given the following dimensions of a settling chamber: diameter = 50 m, depth = 2.5 m, average detention time = 2 hrs. How much quantity of wastewater is being treated [per day]? (Q164, p694; UGC NET Jun 2014).  
<https://www.youtube.com/watch?v=iUITUxD6shE>.

The settling chamber is a cylinder whose volume is  $\pi r^2 h = 3.14 \times 25^2 \times 2.5 = 4906.25 \text{ m}^3$

Detention time = volume  $\div$  flow rate

$\rightarrow$  flow rate = volume  $\div$  detention time =  $2453.125 \text{ m}^3/\text{hour}$  or  $2453.125 \times 24 = 58,874.88 \text{ m}^3/\text{day}$

52. A wastewater treatment plant in a city treats  $50000 \text{ m}^3$  per day. For an average flow rate of  $25 \text{ m}^3$  per day per square meter, what should be the diameter of the tank? (Q179, p696; UGC NET Dec 2013)

Since depth is not given, we shall assume depth to be 1 m. In other words, we are assuming that one cross sectional area of  $1 \text{ m}^2$  will be treating  $25 \text{ m}^3$ . So, to treat  $5 \times 10^4 \text{ m}^3$ , we will need:

$$1 \text{ m}^2 : 25 \text{ m}^3 :: x \text{ m}^2 : 5 \times 10^4 \text{ m}^3$$

$$25x = 5 \times 10^4 \rightarrow x = 2 \times 10^3 \text{ m}^2$$

$$\pi r^2 = \pi (d/2)^2 = \pi d^2/4 = 2 \times 10^3 \rightarrow$$

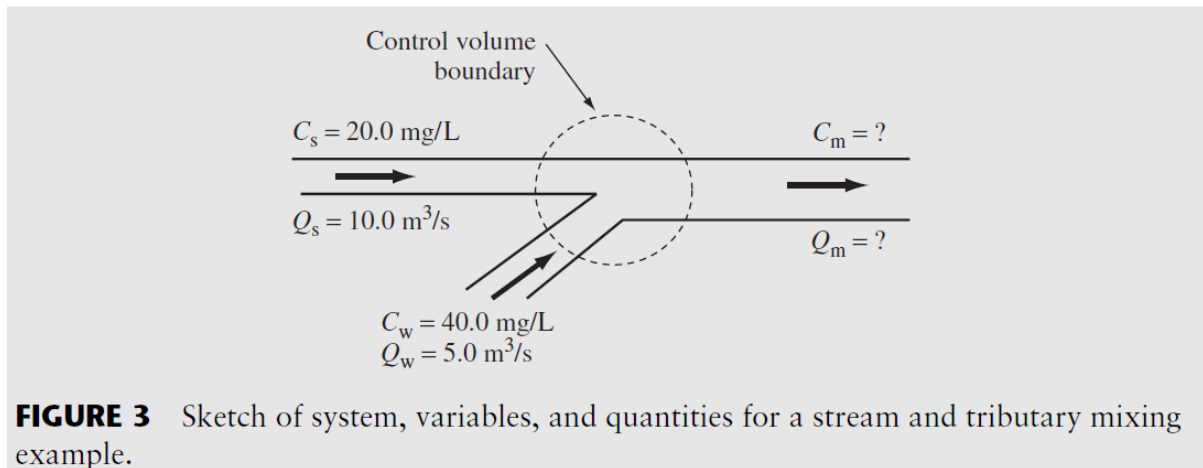
$$d = \sqrt{\frac{4 \times 2000}{22/7}} = 50.452$$

(In the exam, use approximations. So, in the above example, root(2545) was simplified to root(2500) = 50).

### 5.2 Amount of pollutant/BOD mixing into water

(Q172, 152, 123, 11) (see Masters and Ela 2014: 8-9, 211).

53. A stream flowing at  $10 \text{ m}^3/\text{s}$  has a tributary feeding into it with a flow of  $5 \text{ m}^3/\text{s}$ . The stream's concentration of chloride upstream of the junction is  $20 \text{ mg/L}$ , and the tributary chloride concentration is  $40 \text{ mg/L}$ . Find the downstream chloride concentration. (Masters and Ela 2014).



**FIGURE 3** Sketch of system, variables, and quantities for a stream and tributary mixing example.

$$C_m = \frac{C_s Q_s + C_w Q_w}{Q_s + Q_w} = \frac{(20 \times 10) + (40 \times 5)}{(10 + 5)} = 26.7 \text{ mg/L}$$

The same method can be used while measuring dissolved oxygen and BOD.

### 5.3 Biochemical Oxygen Demand

#### 5.3.1 Important formulae

##### **BOD<sub>s</sub> for unseeded samples =**

(Initial Dissolved Oxygen – Dissolved Oxygen after 5 days) ÷ dilution fraction

$$(DO_i - DO_f) \div p$$

Dilution fraction,  $p$  = volume of wastewater ÷ (volume of wastewater + dilution water)

So, BOD<sub>s</sub> for unseeded samples =

$$\frac{(DO_i - DO_f)(\text{volume of wastewater} + \text{dilution water})}{\text{volume of wastewater}}$$

##### **BOD for seeded samples**

In a seeded BOD test, two bottles are prepared. Bottle 1 has wastewater and seeded dilution water to the full capacity of the bottle. Bottle 2 has only seeded dilution water to the full capacity of the bottle.

$$\text{BOD of wastewater} = \frac{(DO_i - DO_f) - (B_i - B_f)(1 - P)}{P}$$

Here,  $DO_i - DO_f$  = change in dissolved oxygen in bottle 1

$B_i - B_f$  = change in dissolved oxygen in bottle 2

$P$  = dilution fraction of wastewater in bottle 1

$$\text{BOD}_m V_m = \text{BOD}_w V_w + \text{BOD}_d V_d$$

## Numerical Questions

$BOD_m$  = BOD of mixture of wastewater and seeded dilution water (bottle 1)

$V_m$  = Volume of the mixture =  $V_d + V_w$

$BOD_w$  = BOD of the wastewater alone

$V_w$  = Volume of wastewater in the mixture (bottle 1)

$BOD_d$  = BOD of the seeded dilution water alone

$V_d$  = Volume of seeded dilution water in the mixture (also bottle 1)

### **BOD as first order reaction**

$$L_t = L_0 e^{-kt}$$

$L_t$  = the amount of oxygen demand left after time  $t$

$L_0$  = Ultimate carbonaceous oxygen demand

$k$  = reaction constant

$$L_0 = BOD_t + L_t$$

$$So, L_0 = BOD_t + L_0 e^{-kt}$$

$$\rightarrow BOD_t = L_0(1 - e^{-kt})$$

To make calculations easier, sometimes,  $e$  is substituted by 10.

$k = 2.303K$ , where  $K$  is reaction rate coefficient to the base 10.

### **Change in reaction rate constant, $k$ , with change in temperature**

$$k_T = k_{20} 1.047^{(T-20)}$$

### **Deoxygenation**

$$\text{Rate of deoxygenation} = k_d L_T$$

Where  $k_d$  = the deoxygenation rate constant (per day)

$L_t$  = the BOD remaining  $t$  days after waste enters river

Since  $L_t = L_0 e^{-k_d t}$ ,

$$\text{Rate of deoxygenation} = k_d L_0 e^{-k_d t}$$

### **Reaeration**

Rate of reaeration = reaeration constant (per unit time)  $\times$  dissolved oxygen deficit =  $k_r D$

## Numerical Questions

$$\text{Reaeration constant, } k_T = \frac{3.9u^{1/2}}{H^{3/2}}$$

$$\text{Initial oxygen deficit of the mixture of river and wastewater, } D_0 = DO_s - \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r}$$

Where,  $D_0$  = initial oxygen deficit of the mixture of river and wastewater

$DO_s$  = saturated value of DO in water at the temperature of the river

$DO_w$  = DO in the wastewater

$DO_r$  = DO in the river just upstream of the wastewater discharge point

$Q_w$  = volumetric flow rate of wastewater

$Q_r$  = volumetric flow rate of river

### 5.3.2 Questions

54. Assume the dilution fraction  $p$  for an unseeded mixture of waste and water is 0.03. the DO of the mixture is initially 9 mg/L and drops to 3 mg/L after five days. The reaction constant  $k$  is 0.22/day. Five-day BOD of water will be? (Q154, p693; UGC NET Jun 2014)

$k$  is given here just to confuse us. It's not needed.

$$BOD_5 = \text{change in BOD in 5 days} \div \text{dilution fraction} = (9-3) \div 0.03 = 200 \text{ mg/L}$$

55. The  $BOD_5$  of wastewater is 150 mg/L at 20 °C. The  $k$  value is 0.23 per day. What will be the ultimate BOD? (Q85, p687; JNU PhD 2016)

$$L_0 = BOD_t + L_0 e^{-kt}$$

Since  $t = 5$  here,

$$L_0(1 - e^{-kt}) = BOD_t$$

$$L_0 = BOD_t / (1 - e^{-kt})$$

Replacing values for  $BOD_5$ ,  $t$  and  $k$ , we get

$$150 / (1 - e^{-0.23 \times 5})$$

56. A 100 ml sample of paper industry effluent digested in acidified dichromate solution and by back titration, it is found that  $5 \times 10^{-4}$  mol of dichromate has been consumed in chemical oxidation. What is COD of the sample? (Q4, p678; UGC NET Jun 2019)

See <https://www.youtube.com/watch?v=le5qYtcAtfQ>

57. A laboratory analysis of a wastewater sample indicated BOD of 750 mg/L and  $k$  of 0.2/day at 20 °C. The five-day BOD will be? (Q111, p517; UGC NET Dec 2015)

## Numerical Questions

$$L_0(1 - e^{-kt}) = \text{BOD}_t$$

Replacing values for  $L_0$ ,  $t$  and  $k$ , we get

$$750(1 - e^{-1}) = \text{BOD}_5$$

Since the calculation involves  $e$ , for some reason

<https://www.youtube.com/watch?v=AFK05wj8FiA> suggests we simply take 10 as its value.

$$\text{This becomes } 750(1 - 10^{-1}) = 675 \text{ mg/L}$$

58. BOD of effluents discharged on land for irrigation should not exceed? (Q118, p517; UGC NET Jun 2010/5)

100 mg/L

59. Water (Prevention and Control of Pollution) Act 1974 allows industries to release waste in inland water having BOD level? (Q317, p534; UGC NET Dec 2009)

30 mg/L

60. The theoretical oxygen demand for a solution containing 500 mg/L of phenol will be? (Q192, p524; UGC NET Sep 2013)

1191 mg/L. How?

*Write other formulae for BOD here*

## 6 Radioactivity and half-life – unit 2, 5 and 6

Radioactivity and half-life (Q263, 251; Q74, p664; Q329, p535)

61. If 10% of a radioactive material decays in 5 days, then the amount of original material left after 20 days is approximately? (Q74, p664; JNU MPhil/PhD 2016) – 60%

After 5 days, 0.10 of the substance has degraded and 0.90 is left.

After 10 days,  $0.9 \times 0.9 = 0.81$  is left.

After 15 days,  $0.9 \times 0.81 = 0.729$  is left, and after 20 days,  $0.9 \times 0.729 = 0.6561 \approx 65\%$  is left.

This can also be solved by  $(0.9)^{20/5} = 0.9^4 = 0.6561$

I don't know how the answer is 60%, when 65% is an answer.

62. After 560 days, 1 gm of a radioactive element is reduced to  $\frac{1}{6}$  g. The half-life of the radioactive element is? (Q329, p535; UGC NET Jun 2009).

According to the question, after 560 days,  $\frac{1}{6}$  g of the substance is left.

Here,  $t_{1/6} = 560$

Let  $x$  be the duration after which the amount of a substance left is  $0.5 = \frac{1}{2}$

The number of  $t_{1/6}$  in  $x$  days is given by  $x/560$ .

$$0.5 = (1/6)^{x/560}$$

I think this can only be solved using a log table. I don't know how to solve it in an exam. The answer is 216 days according to NET.

63. Half life of a radioactive sample is given by? (Q263, p703; UGC NET Dec 2008)

$0.693/\lambda$ , where  $\lambda$  is the decay constant (Smith 2005: 212)

64.  $C_{14}$  has a half life of 5700 years. The fraction of  $C_{14}$  atoms that decay per year is?

The question is essentially asking for decay constant,  $\lambda$ .

$$\lambda = 0.693/t_{1/2} = 0.693/5700 = 12.157 \times 10^{-5}$$

<https://www.youtube.com/watch?v=Kx6SQYfGUcg>

## 7 Solar energy – unit 5

**Solar panel/pond area** (Singh 2019: 657-77: Q6, 28, 75) Solutions from Boeker and van Grondelle 2011: 157)

65. What is the required solar panel area for photovoltaic cells with efficiency of 20% and incident solar radiation of 400 W/m<sup>2</sup> to run an electric geyser of 2kW? (Q6, p657)

Eventual output (here, required output) =  $2 \times 10^3$  W

$$\text{Efficiency} = \frac{\text{output}}{\text{input} \times \text{area}}$$

$$\rightarrow 20\% = \frac{2 \times 10^3}{400 \times \text{area}} \rightarrow 0.2 = \frac{2 \times 10^3}{400 \times \text{area}} \rightarrow \text{area} = \frac{2 \times 10^3}{400 \times 0.2} = 25 \text{ m}^2$$

The problem can also be solved by simple unitary method

1 m<sup>2</sup> is receiving 400 W/m<sup>2</sup> insolation, and its eventual output is 20% of 400 = 0.2 × 400 = 80 W.

If 1 m<sup>2</sup> is producing 80 units of energy, then how many m<sup>2</sup> will be required to produce 2 kW or 2 × 10<sup>3</sup> units? This can be written as:

$$1:80:: x : 2 \times 10^3 \rightarrow 80x = 2 \times 10^3 \times 1 \rightarrow x = (2 \times 10^3) \div 80 \rightarrow x = 25 \text{ m}^2$$

66. A single solar cell on isolation of 800 of W/m<sup>2</sup> produces a voltage of 0.5 V and current of 2 A. The efficiency of solar cell is 12.5%. The area of the cell is?

Eventual output (here, required output) = voltage × current = 1 W

$$\text{Efficiency} = \frac{\text{output}}{\text{input} \times \text{area}}$$

$$\rightarrow 12.5\% = \frac{1}{800 \times \text{area}} \rightarrow 0.125 = \frac{1}{800 \times \text{area}} \rightarrow \text{area} = \frac{1}{800 \times 0.125} = 0.01 \text{ m}^2$$

The problem can also be solved using unitary method.

Total power produced by the solar cell i.e. output is 0.5 × 2 = 1 W.

Total power produced by 1 m<sup>2</sup> of such solar cells = 0.125 × 800

If 1 m<sup>2</sup> of cells are producing 0.125 × 800, then how may m<sup>2</sup> will be required to produce 1 W? The problem can be expressed as:

$$1 : 0.125 \times 800 :: x : 1 \rightarrow 0.125 \times 800 \times x = 1 \rightarrow x = 1/(0.125 \times 800) \rightarrow x = 10^{-2} \text{ m}^2$$

**Energy of a photon** (p657-77: Q17)

67. A photon in the visible blue region irradiates a silicon solar cell. How much energy of photon will be wasted? <https://youtu.be/D9WB9ervcH0>

## Numerical Questions

In order to be absorbed in the semiconducting silicon, the photon must have a minimum of 1.1 eV energy to make the electron conduct. A photon with less energy than this will not be absorbed, and a photon with greater energy than this will only be able to conduct 1.1 eV. The remainder is wasted as heat. A photon in the blue region has energy of 3.1 eV, so 2.0 eV is wasted.

### Efficiency of a solar cell (p657-77: Q35, 143, 163)

68. A single solar cell ( $10\text{ cm} \times 10\text{ cm}$ ) produces a voltage of 0.5 V and a current of 2.5 A. if the solar insolation is  $800\text{ W/m}^2$ , the efficiency of solar cell is? (Q35, p659)

The area of the cell is  $10\text{ cm} \times 10\text{ cm} = 10 \times 10^{-2} \times 10 \times 10^{-2} = 10^{-2}\text{ m}^2$

Output of the cell is  $0.5 \times 2.5 = 1.25\text{ W}$

Also, if  $1\text{ m}^2$  receive 800 units of insolation, then  $10^{-2}\text{ m}^2$  will receive:

1: 800::  $10^{-2} : x \rightarrow x = 8$  units of insolation

Efficiency of the cell is  $1.25 \div 8 = \sim 15.6\%$

69. For a solar flat plate collector the following data is given: useful heat gain =  $28\text{ W/m}^2\text{-hour}$ , solar radiation intensity =  $350\text{ W/m}^2\text{-hour}$  and the factor to convert beam radiation to that on the plane of the collector = 1.2. What is the efficiency of the collector? (Q143, p669)

Output = 28

Input = radiation  $\times$  conversion factor =  $350 \times 1.2 = 420$

Efficiency =  $28 / 420 = 0.0666 = \sim 6.6\%$

70. In case of a photovoltaic cell, the maximum theoretical efficiency of conversion is  **$\sim 45\%$** .

### Photon flux density

71. Solar radiation of average energy  $\sim 1.25\text{ eV}$  and intensity  $500\text{ W/m}^2$  are incident on the surface of a solar flat plate collector. What is the approximate photon flux density impinging on the surface of the collector? (Q77, p664; UGC NET Jul 2016)

Photon flux density is defined as the number of photons per unit area per unit time i.e. no. of photons / sec –  $\text{m}^2$ .

Also, input power ( $\text{W/m}^2$ ) = photon flux density  $\times$  energy (in joules)

The formula is quite similar to Watts (power) = current  $\times$  voltage.

( $1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$ ; so,  $1.25\text{ eV} = 1.25 \times 1.6 \times 10^{-19}\text{ J}$ )

So,  $500 = \text{photon flux density} \times 1.25 \times 1.6 \times 10^{-19}\text{ J}$

Photon flux density =  $2.50 \times 10^{21}\text{ photons/sec-m}^2$ .



<https://www.youtube.com/watch?v=nlAQh4tN4kY>

**Current in a solar cell** (p657-77: Q69, 85)

72. An insolation of  $1000 \text{ W/m}^2$  is incident on a single Si solar cell of area  $80 \text{ cm}^2$ . assuming that 5% of photons create electron-hole pairs and that the average energy of the photons is 1 eV. What is the short circuit current of the cell? (Q69, p663)

If  $1 \text{ m}^2$  receives 1000 units of insolation, then  $80 \text{ cm}^2$  (i.e.  $80 \times 10^{-4} \text{ m}^2$ ) receives -?

$$1:1000 :: 80 \times 10^{-4} : x \rightarrow x = 80 \times 10^{-4} \times 10^3 \rightarrow x = 8 \text{ units}$$

Since efficiency is 5% or 0.05, output will be  $8 \times 0.05 = 0.4$ .

Since output = volts  $\times$  current,  $\rightarrow 0.4 = 1 \text{ eV} \times \text{current} \rightarrow \text{current} = 0.4 \text{ A}$

+ Since we are talking of electron hole *pairs*, 0.4 A will be multiplied by 2  $\rightarrow 0.8 \text{ A}$ .

73. Consider solar insolation of  $400 \text{ W/m}^2$  incident on a single solar cell of area  $100 \text{ cm}^2$ . if only 15% of the photons cause electron-hole pairs and the average energy of incident photons is  $\sim 1 \text{ eV}$ , the short circuit current of the cell is:

If  $1 \text{ m}^2$  is receiving 400 W of insolation, then  $100 \text{ cm}^2$  (or  $0.01 \text{ m}^2$ ) will receive –

$$1:400::0.01:x \rightarrow x = 4 \text{ units}$$

Output = 15% of 4 =  $0.15 \times 4 = 0.6$ .

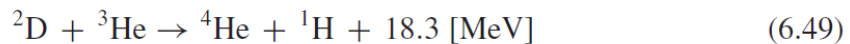
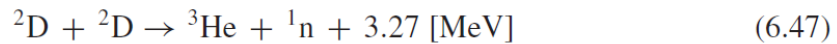
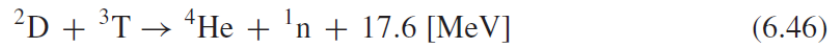
Since output = current  $\times$  voltage  $\rightarrow 0.6 = \text{current} \times 1 \text{ eV} \rightarrow \text{current} = 0.6 \text{ A}$

+ Since we are talking about electron hole *pairs*, 0.6 A will have to be multiplied by 2, which will give us 1.2 A.

## 8 Nuclear energy – unit 5

### Energy (MeV) released in fusion (Q7, 107) (see Boeker and van Grondelle 2011: 238)

From Figure 6.1 it was already concluded that fusion of the lightest nuclei could lead to energy gains. In practice the following reactions are possible:



In these equations deuterium and tritium are indicated by  ${}^2\text{D}$  and  ${}^3\text{T}$ , although the more correct notation would have been  ${}^2\text{H}$  and  ${}^3\text{H}$ .

From Eqs. (6.46) to (6.49) it is clear that the positively charged nuclei need to overcome a Coulomb barrier. (from Boeker and van Grondelle 2011: 238)

### Energy (MeV) released in fission (Q33, 44, 108, 118)

Some essential facts:

Isotopes of hydrogen,

Mass of 1 amu in kg =  $1.66 \times 10^{-27}$  kg

1 eV =  $1.6 \times 10^{-19}$  J

Number of joules energy produced by 1 amu ( $1.66 \times 10^{-27}$  kg) =  $1.5 \times 10^{-10}$  J

Number of MeV energy produced by 1 amu = 931 MeV. (Smith 2004: 206-7)

Number of atoms in 1 kg uranium =  $26 \times 10^{23}$  (Smith 2004: 220-1)

74. In a nuclear fission reaction involving  ${}^{235}\text{U}_{92}$  and a slow neutron, the mass defect is found to be 0.223 u. how much energy will be released from 5 gram of  ${}^{235}\text{U}_{92}$ ? (1u =  $1.66 \times 10^{-27}$  kg) (Q33, p660; UGC NET Jul 2018)

**Method 1** (converting kilogrammes to amu; easier method)

According to the question, 1 atom produces a mass defect of 0.223 amu. 1 amu has the energy of  $1.5 \times 10^{-10}$  J. So, 0.223 amu will have the energy of  $0.223 \times 1.5 \times 10^{-10}$  J.

We also know from Smith (2004: 220-1) that there are  $26 \times 10^{23}$  atoms in 1 kg uranium. So, 5-gram (or, 0.005 kg) uranium will have  $0.005 \times 26 \times 10^{23}$  atoms.

Using simple unitary method,

1 atom :  $0.223 \times 1.5 \times 10^{-10}$  J ::  $0.005 \times 26 \times 10^{23}$  atoms: x joules

## Numerical Questions

$$\rightarrow 0.223 \times 1.5 \times 10^{-10} \times 0.005 \times 26 \times 10^{23} = x \rightarrow x = 0.043485 \times 10^{13} \text{ J} = 434.85 \text{ GJ}$$

This has been simplified by Smith (2004: 221) as:

The energy released by a certain amount of uranium is =

Mass defect (in amu)

× energy contained in 1 amu (i.e.  $1.5 \times 10^{-10}$  J)

× mass of uranium (in kilogrammes) given (in the question)

× number of atoms in 1 kg uranium ( $26 \times 10^{23}$ )

If, instead of the mass defect, the energy released by 1 atom is given straightaway, then the formula is simplified even further:

Energy released by 1 atom (in eV or J)

× mass of uranium (in kilogrammes) given (in the question)

× number of atoms in 1 kg uranium ( $26 \times 10^{23}$ )

**Method 2** (converting atoms to kilogrammes. A bit unnecessarily convoluted, but no reason why the logic is untenable.)

According to the question, 1 atom produces a mass defect of 0.223 amu. 1 amu has the energy of  $1.5 \times 10^{-10}$  J. So, 0.223 amu will have the energy of  $0.223 \times 1.5 \times 10^{-10}$  J.

Also, one uranium atom is 235.1 amu. Since  $1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$ , the weight of one uranium atom in kg is  $235.1 \times 1.66 \times 10^{-27} \text{ kg}$ .

Using unitary method, if 1 atom produces  $0.223 \times 1.5 \times 10^{-10}$  J, 5 g (or, 0.005 kg) would produce  $x$  joules energy.

$$235.1 \times 1.66 \times 10^{-27} : 0.223 \times 1.5 \times 10^{-10} :: 5 \times 10^{-3} : x$$

$$\rightarrow x = (0.223 \times 1.5 \times 10^{-10} \times 5 \times 10^{-3}) / (235.1 \times 1.66 \times 10^{-27})$$

$$= 428.554 \times 10^9 \text{ J} = 428.554 \text{ GJ}$$

### Conclusion

What I do not understand is how the answer using both methods is different. And none of them exactly match the answer given by UGC i.e. 426.7 GJ.

75. A typical fission reaction involving an atom of  $^{235}\text{U}_{92}$  leads to a mass defect of  $0.37 \times 10^{-27} \text{ kg}$ . how much energy is going to be released by 1 g of  $^{235}\text{U}_{92}$ ? (Q44, p661; UGC NET Nov 2017)

**Method 1** (straightforward method, but involves a bit heavy calculations)

The weight of 1 amu is  $1.66 \times 10^{-27} \text{ kg}$ . So, how many amu will be contained in  $0.37 \times 10^{-27} \text{ kg}$ .

$$1 : 1.66 \times 10^{-27} \text{ kg} :: x : 0.37 \times 10^{-27} \text{ kg} \rightarrow x = 0.37/1.66 \text{ amu}$$

Applying the above equation for energy released by a certain amount of uranium,

Mass defect (in amu)

× energy contained in 1 amu (i.e.  $1.5 \times 10^{-10}$  J)

× mass of uranium (in kilogrammes) given in the question (i.e.  $10^{-3}$  kg)

× number of atoms in 1 kg uranium ( $26 \times 10^{23}$ )

We get,  $(0.37/1.66) \times 1.5 \times 10^{-10} \times 10^{-3} \times 26 \times 10^{23} = 8.692 \times 10^{10}$  J.

**Method 2** (easier method, involving less calculations)

1 atom is releasing energy equivalent to:

$$E = mc^2$$

$$E = 0.37 \times 10^{-27} \times (3 \times 10^8)^2 = 3.33 \times 10^{-11} \text{ J}$$

Also, 1 kg uranium contains  $26 \times 10^{23}$ ; so,  $10^{-3}$  kg will contain  $26 \times 10^{23} \times 10^{-3}$  atoms.

So, 1 g is releasing  $3.33 \times 10^{-11} \times 26 \times 10^{23} \times 10^{-3} \text{ J} = 8.658 \times 10^{10} \text{ J}$  energy.

<https://www.youtube.com/watch?v=DuhNdxuW5pI>

76. Fast neutrons produced in a nuclear fission reaction involving  $^{235}\text{U}_{92}$  have average energy in the order of? (Q108, p667; UGC NET Jun 2015)

Ans: 2 MeV

77. If fission of 1 atom of  $^{235}\text{U}$  produces 200 MeV energy, how much energy will be produced by 1 metric tonnes of  $^{235}\text{U}$ ? (Q118, p667; UGC NET Dec 2014)

Using above formula,

Energy released by 1 atom (in eV or J)

× mass of uranium (in kilogrammes) given (1 metric tonne = 1000 kg)

× number of atoms in 1 kg uranium ( $26 \times 10^{23}$ )

$$200 \times 10^6 \times 10^3 \times 26 \times 10^{23} = 52 \times 10^{34} \text{ eV}$$

We know that  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$\text{So, } 52 \times 10^{34} \text{ eV} = 52 \times 10^{34} \times 1.6 \times 10^{-19} = 8.32 \times 10^{15} \text{ J}$$

**Number of neutrons produced in a reaction (Q135)**

78. In a nuclear fusion reactor it is envisaged to use a liquid blanket of Li to absorb fast neutrons from D+T reaction. How many neutrons are produced as a result of  ${}^7_3\text{Li} + {}^1_0\text{n}^1$ ? (Q135, p669; UGC NET Jun 2014)

The answer is 2 neutrons.

I am no physicist, but it seems it is because the D + T reaction produces one neutron; and when the neutron hits the fission reaction blanket ( $\text{Li} + \text{n}$ ), the fission reaction gives rise to another neutron. So, a total of two neutrons.

**Kind of particles released (Q84)**

79. Disintegration of  $^{226}\text{Ra}_{88}$  yields  $^{222}\text{Rn}_{86}$  owing to the emission of: (a) two  $\beta$  particles (b) one  $\alpha$  particle (c) gamma radiation (d) one  $\beta$  particle followed by an  $\alpha$  particle (Q84, p665; UGC NET Jul 2016)

Each time atomic mass decreases by 4 and atomic number by 2, it's an  $\alpha$  decay i.e. an  $\alpha$  particle (a Helium nucleon) has been emitted.

If the mass stays the same and the atomic number increases by 1, it's a  $\beta$  decay i.e. a  $\beta$  particle (an electron) has been emitted.

The example in the question is that of an  $\alpha$  emission.

Also read Masters and Ela (2014: 76-81).

## 9 Magnetohydrodynamic power – unit 5

### Power from an MHD plant (Q9, 156, 174)

80. Calculate the open circuit voltage and maximum power output for the following MHD generator:

$$\text{Plate area (A)} = 0.25 \text{ m}^2$$

$$\text{Distance between plates (d)} = 0.50 \text{ m}$$

$$\text{Magnetic flux density (B)} = 2 \text{ Wb/m}^2$$

$$\text{Average gas velocity (U)} = 10^3 \text{ m/s}$$

$$\text{Gas conductivity } (\sigma) = 10 \text{ mhos/m } (\text{https://www.engineeringenotes.com/essay/power-generation/essay-on-mhd-power-generation-electricity-energy-management/20355})$$

$$\text{Voltage (E)} = B \times U \times d = 10^3 \text{ V}$$

$$\text{Resistance (R)} = d/(\sigma A) = 0.50 / (10 \times 0.25) = 0.2 \text{ ohms}$$

$$\text{Maximum power, } W_{max} = \text{Voltage}^2 \div 4R = 10^6 \div 0.8 = 1250 \text{ kW}$$

$$\text{A simpler formula for } W_{max} = (B^2 \times U^2 \times d \times \sigma \times A)/4 = \mathbf{0.25 \times B^2 \times U^2 \times d \times \sigma \times A}$$

81. In an ideal MHD power plant, the velocity of fuel gas is increase by a factor of 1.5 and the magnetic field is increased by a factor of 2. By what factor will the maximum power ( $W_{max}$ ) will increase? (Q9, p657)

$$\text{Using } W_{max} = 0.25 \times B^2 \times U^2 \times d \times \sigma \times A,$$

$$W_{max} \text{ initial} = 0.25 \times B^2 \times U^2 \times d \times \sigma \times A$$

$$W_{max} \text{ final} = 0.25 \times (2B)^2 \times (1.5U)^2 \times d \times \sigma \times A$$

$$= 0.25 \times 4B^2 \times 2.25U^2 \times d \times \sigma \times A = 4 \times 2.25 \times 0.25 \times B^2 \times U^2 \times d \times \sigma \times A$$

$$= 9 \times W_{max} \text{ initial}$$

### Voltage developed across plates

82. In an ideal MHD power generator, the separation between plates = 0.5, magnetic flux density = 2 Wb/m<sup>2</sup> and average fuel velocity is 500 mps. The voltage developed across plates is (Q125, p668)

$$\text{Voltage} = \text{distance between plates} \times \text{magnetic flux density} \times \text{average fuel velocity}$$

$$= 0.5 \times 2 \times 500 = 500 \text{ V}$$

**Efficiency of an MHD plant** (Q161, 136, 174, 215) is 50% or 0.5. The efficiency of an MHD plant is expressed as  $\frac{|\text{Voltage (E)}|}{|\text{Velocity of hot ionized gas (U)}| |\text{Magnetic Field (B)}|}$

## 10 Wind energy – unit 5

Power output from windmill (Q11, 151, 165)

83. A windmill has 10 m diameter and efficiency of 40%. Wind velocity at normal temperature/pressure is 10 mps and the density of air is 1.29 kg/m<sup>3</sup>. The expected power output from the windmill is? (Q11, p657; UGC NET June 2019)

Power output of the windmill =

$$\frac{1}{2} \times \text{Area of windmill} \times \text{density of air} \times \text{wind velocity}^3 \times \text{power coefficient}$$

$$= \frac{1}{2} \times A \times \rho \times v^3 \times C_p \text{ (Smith 2004: 55)}$$

$$\frac{1}{2} \times (\pi \times (10/2)^2) \times 1.29 \times 10^3 \times 0.4 = 20,394.3 \text{ W or } 20.3943 \text{ kW}$$

84. Consider a windmill with the following parameters: cross sectional area = 30 m; wind speed = 10 mps; density of air = 1.29 kg/m and conversion efficiency = 0.4. What is the power output of the windmill? (Q151, p670; UGC NET Sep 2013)

Power output of the windmill =

$$\frac{1}{2} \times \text{Area of windmill} \times \text{density of air} \times \text{wind velocity}^3 \times \text{power coefficient}$$

$$= \frac{1}{2} \times 30 \times 1.29 \times 10^3 \times 0.4 = 7740 \text{ W or } 7.740 \text{ kW}$$

85. A windmill has cross sectional area of 25 m<sup>2</sup> and wind speed is 6 mps. What's the power output at Betz limit? (Q59, p662; UGC NET Jan 2017)

Power output of the windmill =

$$\frac{1}{2} \times \text{Area of windmill} \times \text{density of air} \times \text{wind velocity}^3 \times \text{power coefficient}$$

In this case, power coefficient is the Betz limit

Since density of air is not given, we shall take it to be 1.29 kg/m<sup>3</sup>.

$$\frac{1}{2} \times 25 \times 1.29 \times 6^3 \times (16/27) = \frac{1}{2} \times 25 \times 1.29 \times 216 \times (16/27)$$

$$= 25 \times 1.29 \times 108 \times (16/27) = 2700 \times 1.29 \times (16/27) = 100 \times 1.29 \times 16$$

$$= 2064 \text{ W} = 2.064 \text{ kW}$$

### Ratio/% of power output increase

86. If the wind speed in an ideal wind turbine changes by 5%, by what percentage will the electrical power change? (Q83, p665; UGC NET Jul 2016)

The power output of a windmill changes with the third power of the wind velocity (Boeker and van Grondelle 2011: 159).

$$\text{Initial power output (P}_i\text{)} = \frac{1}{2} A \rho v^3$$

## Numerical Questions

$$\text{Final power output } (P_f) = \frac{1}{2} A\rho(1.05v)^3$$

$$\text{Percentage change} = \{(P_f - P_i)/P_i\} \times 100$$

$$= \frac{\frac{1}{2}A\rho(1.05v)^3 - \frac{1}{2}A\rho v^3}{\frac{1}{2}A\rho v^3} \times 100 = \frac{\frac{1}{2}A\rho v^3(1.05^3 - 1^3)}{\frac{1}{2}A\rho v^3} \times 100$$

$$= (1.157625 - 1) \times 100 = 15.7625\%$$

<https://www.youtube.com/watch?v=VFRQq1Bmbyk>

87. Consider an ideal wind whose wind speed increases by 3 times. The power output will increase by how many times? (Q159, p671; UGC NET Jun 2013)

Remember, that unlike the previous question, it is not asking for the percentage change but for the ratio by which the power output changes.

$$\text{Initial power output } (P_i) = \frac{1}{2} A\rho v^3$$

$$\text{Final power output } (P_f) = \frac{1}{2} A\rho(3v)^3 = 27P_i$$

$$\text{Ratio change} = P_f/P_i = 27$$

### Coefficient of performance for the windmill ( $C_p$ )

88. If  $a$  is the fractional wind speed decrease at the wind turbine, the maximum extraction of power from the wind occurs when  $a$  is equal to? (Q165, p671; UGC NET Dec 2012)

It has been found out that power output is maximum when  $a$  is  $1/3$ . In other words, the power output is maximum when the velocity of outflowing air ( $v_{out}$ ) is  $1/3$  of velocity of inflowing air ( $v_{in}$ ) i.e.  $v_{out} = v_{in}/3$

89. If the fractional wind velocity drop at the rotor of a windmill is 0.1, the coefficient of performance of the windmill is? (Q21, p658; UGC NET Dec 2018)

The fractional wind velocity drop at the rotor of a windmill is simply the decrease in the speed of the wind after it passes through the windmill. Let it be represented by  $a$ .

In such a case, coefficient of performance of a windmill ( $C_p$ ) is given by  $4a(1-a)^2$ .

Substituting values, we get  $(4 \times 0.1) (1 - 0.1)^2 = 0.4 \times 0.9^2 = 0.4 \times 0.81 = 0.324$

(Boeker and Grondelle 2011: 161; <https://www.youtube.com/watch?v=QDQvygWn-5E>)

90. The maximum coefficient of performance of a wind turbine is? (Q175. 188, 216) (UGC NET Dec 2012, Dec 2011, Dec 2007)

The maximum coefficient of performance of a wind turbine, aka Betz Limit = 0.59 or  $16/27$  (Boeker and van Grondelle 2011: 161).

**Range of wind speeds for windmill (Q179) – Most suitable wind range speed is 4-12 mps.**



## 11 Coal power – unit 5

### Amount of SO<sub>2</sub> production (Q12, 61)

91. A 500 MW coal-based power station is operating at an efficiency of 30%. If the coal has 1% of Sulphur content and 1 tonne of coal produces 8000 kWh energy, how much SO<sub>2</sub> will be emitted daily by the plant? (Q12, p658; UGC NET Dec 2018)

Efficiency is always output/input.

In this case, the output is  $500 \times 10^6$  W, and the efficiency is 0.30. So, input is  $(500 \times 10^6)/0.30 = 1666.66 \times 10^6$  W

Since the unit watts (W) means joules per second (J/s), we can say that  $1666.66 \times 10^6$  joules are being used in a second. Since there are 86400 seconds in a day, we can say that the coal power plant is getting an input of  $1666.66 \times 10^6 \times 86400$  joules ( $= 144 \times 10^{12}$  J) in a day.

Next, we know that 1 tonne of coal produces 8000 kWh of energy.

Power (Watts) = Energy (Joules) / Time (seconds) i.e. power is the rate at which energy is produced/consumed (<https://www.energylens.com/articles/kw-and-kwh>).

So, Energy (J) = Power (W)  $\times$  Time (seconds)

So, 8000 kWh of energy is  $8000 \times 3600$  joules (1 hr = 3600 secs)

$= 288 \times 10^8$  joules.

1 tonne coal produces 8000 kWh or  $288 \times 10^8$  joules. So, in order to produce  $144 \times 10^{12}$  J, we need  $x$  tonnes of coal.

$1 : 288 \times 10^8 :: x : 144 \times 10^{12}$

$\rightarrow 288 \times 10^8 x = 144 \times 10^{12} \rightarrow x = (144 \times 10^{12}) / (288 \times 10^8) = 5 \times 10^3$  tonnes

This coal, according to the question, has 1% sulphur. So,  $5 \times 10^3$  tonnes would contain (1% of  $5 \times 10^3$ ) 50 tonnes.

The molecular weight of SO<sub>2</sub> is 64 units (32 + (2  $\times$  16)). If 32 units of sulphur are producing 64 units of SO<sub>2</sub>, then 50 units would produce 64 units of SO<sub>2</sub>.

<https://www.youtube.com/watch?v=MLHgdnu53M8>

92. A certain type of coal is burnt at the rate of 0.5 kg/s. If the sulphur content of coal is 5% and 5% of sulphur remains in ash, how much sulphur dioxide is formed per day? (Q61, p663; UGC NET Aug 2016)

If 0.5 kg is consumed in a second, then one day consumes  $0.5 \times 86400$  kg.

Of these  $0.5 \times 86400$  kg, 5% (or 0.05) is sulphur. So, total sulphur content can be expressed as =

## Numerical Questions

$$0.5 \times 86400 \times 0.05 \text{ kg.}$$

Of this  $0.5 \times 86400 \times 0.05 \text{ kg}$ , 5% remains in ash, which means 95% is released in the atmosphere. The amount released into the air can be expressed as =

$$0.5 \times 86400 \times 0.05 \times 0.95 \text{ kg.}$$

Since the molecular weight of  $\text{SO}_2$  is 64 kg, and that of sulphur is 32 kg; it can be concluded that 32 kg of sulphur produces 64 kg of  $\text{SO}_2$  (i.e. double the amount).

So,  $0.5 \times 86400 \times 0.05 \times 0.95 \text{ kg}$  produces

$$0.5 \times 86400 \times 0.05 \times 0.95 \times 2 \text{ kg SO}_2 = 432 \times 9.5 = 4104 \text{ kg/day SO}_2.$$

Tip: Multiplying any three-digit number by 9.5 will take quite some time in an exam. So, just multiply it by 9, and select the number just higher among the options list.

### Amount of ash production (90)

93. A thermal power plant based on coal produces 100 MW of electrical power with the conversion efficiency of 30%. If the ash content of coal is 5% and net heating value is 30 MJ per kg, how much ash is produced daily?

$$\text{Efficiency} = \text{output} / \text{input} \rightarrow \text{input} = \text{output} / \text{efficiency} \rightarrow (100 \times 10^6) / 0.3 \text{ W} = 10^9 / 3 \text{ Watts}$$

Since watts are joules per second (J/s), the amount of input (in joules) in a day will be =  $(10^9 / 3) \times 86400$  joules (in a day).

The net heating value is 1 kg to 30 MJ or  $30 \times 10^6 \text{ J}$ . So,  $\{(10^9 / 3) \times 86400\}$  would correspond to, let's say,  $x$  kgs.

$$1 : 30 \times 10^6 :: x : \{(10^9 / 3) \times 86400\} \rightarrow 30 \times 10^6 x = \{(10^9 / 3) \times 86400\}$$

$$\rightarrow x = \{(10^9 / 3) \times 86400\} / \{30 \times 10^6\} = 96 \times 10^4 \text{ kgs.}$$

Of this, 5% (or, 0.05) is ash. So,  $0.05 \times 96 \times 10^4 \text{ kgs} = 48 \times 10^3 \text{ kgs}$

= 48 metric tonnes

(1 metric tonne = 1000 kilogrammes)

## 12 Tidal power – unit 5

### Potential energy available for tidal period (Q43, 70, 109)

94. A tidal power station has basin area  $10000 \text{ m}^2$  and water trapped at height 2 m above low tide. If the density of sea water is  $1025 \text{ kg/m}^3$ , the potential energy available for every tidal period is (Q43, p661; UGC NET Nov 2017)

Energy available per tidal cycle =  $\frac{1}{2} \times \text{density} \times \text{area} \times \text{height}^2 \times \text{gravitational acceleration}$  (Smith 2004: 57), written as  $\frac{1}{2}\rho h^2 g$

Substituting values, we get

$$\frac{1}{2} \times 1025 \times 10^4 \times 2^2 \times 9.8 = 20090 \times 10^4 \text{ J} = 200.9 \times 10^6 \text{ J}$$

$$= 200.9 \text{ MJ or approx. 201 MJ}$$

<https://www.youtube.com/watch?v=FWrYuPKtSx8>

95. A tidal estuary of tidal range 5 m has an area of water trapped at  $10 \text{ km}^2$ . If we assume density of water  $10^3 \text{ kg/m}^3$  and  $g = 9.8 \text{ mps}^2$ , what is the maximum theoretical energy available per cycle? (Q70, p663; UGC NET Aug 2016)

Energy available per tidal cycle =  $\frac{1}{2} \times \text{density} \times \text{area} \times \text{height}^2 \times \text{gravitational acceleration}$  (Smith 2004: 57). Remember that it's  $10 \text{ km}^2$ , which is equal to  $10 \times 10^6 \text{ m}^2$ .

Substituting values, we get

$$\frac{1}{2} \times 10^3 \times 10 \times 10^6 \times 5^2 \times 9.8 = 12.25 \times 10^{11} \text{ J}$$

96. A tidal estuary of tidal range 2 m has trapped water area  $5000 \text{ m}^2$ . If  $g = 9.8 \text{ mps}^2$ , the maximum theoretical energy available per tidal cycle is?

Energy available per tidal cycle =  $\frac{1}{2} \times \text{density} \times \text{area} \times \text{height}^2 \times \text{gravitational acceleration}$  (Smith 2004: 57), written as  $\frac{1}{2}\rho h^2 g$ . Since density is not given here, we will assume it to be  $10^3 \text{ kg/m}^3$

Substituting values, we get

$$\frac{1}{2} \times 10^3 \times 5000 \times 2^2 \times 9.8 = 98 \times 10^6 \text{ J} = 98 \text{ MJ.}$$

In order to convert Joules of energy to power, divide Joules (energy) by seconds of time.

[https://www.youtube.com/watch?time\\_continue=95&v=YO8Q2L\\_xTs&feature=emb\\_title](https://www.youtube.com/watch?time_continue=95&v=YO8Q2L_xTs&feature=emb_title)

So, 98 MJ would be equal to 98 Megawatts (MW) if one cycle is equivalent to 1 second (is it?). So, I think the answer given in the book i.e. 98 kW is wrong. It should be 98 MW. <https://www.youtube.com/watch?v=6UWs7l-HoPo>

## 13 Dam – unit 5

### Rate of falling water on turbines

97. A 2000 MW dam has maximum head of 200 m. What is the rate of falling water on the turbines? (Neglect losses) (Q50, p661; UGC NET Jan 2017)

The rate of falling water in a dam is measured by volumes per second (litres per second or  $\text{m}^3/\text{s}$ ). Power or potential energy of a dam is given by

$$= mgh = \text{mass (in this case, mass of water)} \times \text{gravity} \times \text{height (aka head)}$$

Since mass = density  $\times$  volume,

potential energy of a dam = density  $\times$  volume  $\times$  gravity  $\times$  height.

By default, we take the density of water to be  $10^3 \text{ kg/m}^3$

So, using the above equation,

$$\text{Power (2000 MW)} = 10^3 \times \text{volume} \times 9.8 \times 200$$

$$= 2000 \times 10^6 \text{ W} = 10^3 \times \text{volume} \times 9.8 \times 200 \rightarrow \text{volume} = 104 \div 9.8 = \sim 1020.41 \text{ m}^3/\text{s}$$

<https://www.youtube.com/watch?v=V8kkoV18mbk>

98. A hydropower plant has a reservoir storage capacity of  $2 \times 10^3 \text{ m}^3$  and its maximum head is 500 m. what is the potential energy? (UGC NET Jan 2017)

potential energy of a dam = density (in this case, of water)  $\times$  volume  $\times$  gravity  $\times$  height

$$= 10^3 \times (2 \times 10^3) \times 9.8 \times 500 = 9.8 \times 10^9 \text{ Joules or } 9.8 \text{ gigajoules}$$

<https://www.youtube.com/watch?v=inysm-N3F04>

## 14 Ocean waves – unit 5

### Power available per meter (Q58, p662; Q66, p663)

99. An ocean wave of 2m height has a time period of 10 sec. Approx power associated with per meter of wavefront is? (Q66, p663; UGC NET Aug 2016)

starts to be dissipated as soon as the wave feels the bottom. The energy of a deep water wave expressed as power in watts per metre of wavefront can be shown to be given by:

$$P = \frac{\rho g^2 H^2 T}{32\pi} \quad (2.11)$$

where  $\rho$  is the density of water, which for sea water is  $1,025 \text{ kg m}^{-3}$  and  $H$  is the height of the wave. To a reasonable approximation this simplifies to:

$$P \approx H^2 T \quad (2.12)$$

where  $P$  is the power in kW per metre of wavefront,  $H$  is the height in metres and  $T$  is the time period in seconds. So for a typical Atlantic swell of 3 m with a 9 s time period, this gives  $81 \text{ kW m}^{-1}$ , which makes it

(from Smith 2004: 58)

Using  $P \approx H^2 T$ , we get  $2^2 \times 10 \approx 40 \text{ kW/m}$

<https://www.youtube.com/watch?v=3Kns7G9xGkE>

100. Consider ocean waves of amplitude 2 m and wavelength 200m. assuming the gravity to be the only active force, the power available per metre perpendicular to the propagation of the wave is?

It is difficult to calculate the power, as neither the velocity nor time period of the wave are given. A (possible) solution, however, for the above question is given in MindMapping's video <https://www.youtube.com/watch?v=3Kns7G9xGkE>. But it's too complicated for a 2 minute question, and is most probably even incorrect; so, skipping it here.

## 15 Energy consumption – unit 5

101. Suppose the world's energy consumption per year grows exponentially at a growth rate of 2% per year. In how much time will the energy consumption per year be double of the present value (Q72, p664; UGC NET Aug 2016)?

$$dN/dt = rN$$

$$dN = 2N - N = N$$

$$r = 0.02$$

Substituting these values, we get  $N/dt = 0.02N \rightarrow dt = 1/0.02 = 50$  yrs.

I don't know how the answer is 35 yrs.

## 16 Fuel cells – unit 5

Voltage and efficiency (Q132, 144) (Boeker and van Grondelle 2011: 120-2)

102. In a methane fuel cell, what will be the voltage of the cell and its efficiency? ( $\Delta G^\circ = 8 \times 10^5$  Joules/gm; mole  $\Delta H^\circ = 8.8 \times 10^5$  Joules/gm; mole and faraday's constant = 96500 coulomb/gm-mole). (Q132, p668; UGC NET Jun 2014)

The efficiency of a fuel cell is given by  $\Delta G^\circ/\Delta H^\circ = (8 \times 10^5)/(8.8 \times 10^5) = 90.9\%$

The voltage in a fuel cell is given by

$$= \frac{\Delta G^\circ}{\text{number of electrons involved in the reaction} \times \text{Avogadro number} \times \text{Number of joules in 1 eV}}$$

The number of electrons involved in this reaction ( $2\text{O}_2 + 8\text{H}^+ + 8\text{e}^- \rightarrow 4\text{H}_2\text{O}$ ) are 8. Avogadro number is  $6.023 \times 10^{23}$ . 1 eV has  $1.602 \times 10^{-19}$  joules.

Substituting these values, we get

$$\frac{8 \times 10^5}{8 \times 6.023 \times 10^{23} \times 1.602 \times 10^{-19}} = 1.036 \sim 1.04 \text{ volts}$$

<https://www.youtube.com/watch?v=WNVaHJYjAN0>

103. For the reaction in a hydrogen oxygen fuel cell,  $\text{H}_2 + \frac{1}{2} \text{O}_2 \rightarrow \text{H}_2\text{O}$ . ( $\Delta G^\circ = 240$  Joules/gm; mole of  $\text{H}_2$  and faraday's constant = 96500 coulomb/gm-mole). (Q144, p669; UGC NET Dec 2013)

The voltage in a fuel cell is given by

$$= \frac{\Delta G^\circ}{\text{number of electrons involved in the reaction} \times \text{Avogadro number} \times \text{Number of joules in 1 eV}}$$

The number of electrons involved in this reaction, i.e. in a hydrogen fuel cell ( $\text{H}_2 + \frac{1}{2} \text{O}_2 + 2\text{e}^- \rightarrow \text{H}_2\text{O}$ ) are 2. Avogadro number is  $6.023 \times 10^{23}$ . 1 eV has  $1.602 \times 10^{-19}$  joules. (<https://www.youtube.com/watch?v=-oGF7kIbtqI>).

Substituting these values, we get

$$\frac{240}{2 \times 6.023 \times 10^{23} \times 1.602 \times 10^{-19}} = 1.2436 \text{ eV}$$

## 17 Aquifers – unit 4

### Rate of flow of water (Q198)

104. A confined aquifer of thickness 25 m has two wells 200 m apart along the direction of flow of water. The difference in their hydraulic heads is 1 m. if hydraulic conductivity is 50 m/day, the rate of flow of water per day per metre of distance perpendicular to the flow of water is? (Q 198, p634; UGC NET Dec 2013)

This question can be solved using Darcy's Law, which states that

Rate of flow of water

= (hydraulic conductivity ( $k$ )  $\times$  thickness of aquifer  $\times$  difference in hydraulic heads ( $dh$ ))/ distance between the two wells ( $dl$ )

$$= (50 \times 25 \times 1)/200 = 6.25 \text{ m}^3/\text{day}$$

<https://www.youtube.com/watch?v=f4d73mrTZ30>

### Specific yield or effective porosity (Q121, 186, 213) (see Masters and Ela 2014: 233 for solutions)

105. An aquifer of sand has a saturated column of cross-sectional area  $0.2 \times 5$  m and a depth of 3 m. if the specific yield of sand is 25%, how much water can be extracted from the aquifer? (Q121, p627; UGC NET Dec 2015)

The volume of the aquifer is  $0.2 \times 5 \times 3 = 3 \text{ m}^3$

Extractable volume from aquifer = specific yield  $\times$  volume =  $3 \times 0.25 = 0.75 \text{ m}^3$

<https://www.youtube.com/watch?v=5UYjQOmUx70>

106. For an aquifer of gravel having cross sectional area of  $4 \text{ m}^2$  and a depth of 2.5 m, how much water could potentially be extracted? (The porosity and specific yield of gravel are 25% and 20% respectively. (Q186, p633; UGC NET Jun 2014)

According to the formula above,

Extractable volume from aquifer = specific yield  $\times$  volume

$$= 10 \text{ m}^3 \times 0.20 = 2 \text{ m}^3$$

Actual volume of water stored in aquifer = porosity  $\times$  volume =  $0.25 \times 10 \text{ m}^3 = 2.5 \text{ m}^3$

Note:  $2 \text{ m}^3$  is the correct answer according to UGC, which can only be arrived at if we multiply volume by specific yield. This is also the answer given by

<https://www.youtube.com/watch?v=8IGTNTxDx1M> using the same formula.

Thus, porosity seems to be redundant here – and that makes sense, because specific yield itself is a function of porosity (specific yield = porosity – specific retention; [http://www.aqtesolv.com/aquifer-tests/aquifer\\_properties.htm](http://www.aqtesolv.com/aquifer-tests/aquifer_properties.htm)). Porosity might have



been helpful if we had to calculate the actual amount of water stored (i.e. volume  $\times$  porosity = 2.5 m<sup>3</sup>).

Nonetheless, there is a video <https://www.youtube.com/watch?v=StEM0q3A8OU> that uses the formula  $volume \times porosity \times specific\ yield$ . Using that formula, the video arrives at 0.5 m<sup>3</sup> as the answer, which is an incorrect answer according to UGC and is not even there in the options list. The formula also does not make sense, because, as I said, specific yield itself is a function of porosity. So, there is no need to multiply the specific yield by porosity.

107. What is the estimate of the volume of water yield for saturated pond aquifer of 1 m width and 2 m depth and 4 m length. Consider the porosity of sand to be 35% and specific yield to be 25%. (Q213, p635; UGC NET Jun 2013)

Using the above formula, the answer to the question should be 2 m<sup>3</sup>. even if we multiply that by the porosity (0.35), the answer is 0.7 m<sup>3</sup>. Neither 2 m<sup>3</sup> nor 0.7 m<sup>3</sup> are the correct answers according to UGC, nor is there any YouTube video for the question. So, I don't know what the right answer is.

### Transmissivity of the aquifer (Q133)

108. If the hydraulic conductivity is 0.2 m per day and thickness of the aquifer is 1.5 m, the transmissivity of the aquifer will be? (Q133, p628; JNU PhD exam)

Transmissivity = hydraulic conductivity  $\times$  thickness = 0.3 m<sup>2</sup>/day

<http://www.aqtesolv.com/aquifer-tests/glossary-of-aquifer-testing-terms.htm#Transmissivity>

Some important formulae:

Flow rate (Q) = hydraulic conductivity aka permeability coefficient ( $k$ , m/day)  $\times$  cross sectional area ( $A$ , m<sup>2</sup>)  $\times$  hydraulic gradient ( $dh/dl$ )

Darcy velocity ( $v$ ) = Flow rate (Q)  $\div$  Cross sectional area (A)

= hydraulic conductivity ( $k$ )  $\times$  hydraulic gradient ( $dh/dl$ )

Seepage velocity aka average linear velocity = Darcy velocity  $\div$  porosity

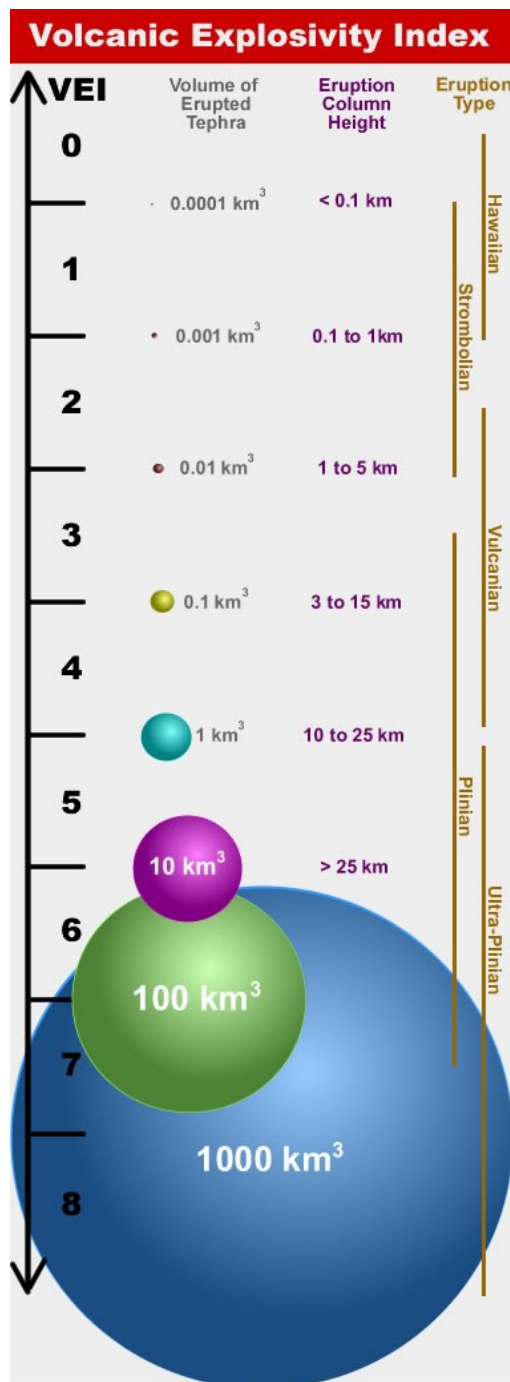
= (Darcy velocity  $\times$  total volume)  $\div$  porosity

## 18 Volcanoes – unit 4

Volcanic explosivity index value (Q193, p634)

109. The volume of ejecta and the column height for a volcano are  $10^{8.5} \text{ m}^3$  and 24 km respectively. What is the volcanic explosivity index value? (Q193, p634; UGC NET Dec 2013)

Volcanic Explosivity Index (VEI) is a relative scale that enables explosive volcanic eruptions to be compared with one another. Ejecta volume is  $10^{0.5} \text{ km}^3$  and column height is 24 km. According to the diagram below (<https://geology.com/stories/13/volcanic-explosivity-index/>), the correct answer is 4 or 5. Since 5 is not an option in the list, we will select 4.



## 19 CFC number – unit 10

110. **Calculate the CFC number** for the halocarbon  $\text{CCl}_2\text{F}_2\text{CF}_3$  (Q7, p768; UGC NET DEC 2018)

To find the number, given the chemical formula: consider the number as consisting of 3 digits: a, b, and c. For 2-digit numbers (e.g., CFC-11) the 'a' digit is zero (e.g., CFC-011).

a is the number of carbon atoms minus 1;

b is the number of hydrogen atoms plus 1;

c is the number of fluorine atoms.

For  $\text{CCl}_2\text{F}_2\text{CF}_3$ :

a = the number of carbon atoms minus 1 =  $2 - 1 = 1$

b = the number of hydrogen atoms plus 1 =  $0 + 1 = 1$

c = the number of fluorine atoms = 5

and, the compound is CFC-115. <https://cdiac.ess-dive.lbl.gov/pns/cfcinfo.html>.

## 20 Probability (unit 9, p752-766)

111. A container has 3 white balls and 2 black balls identical in size. Two balls are drawn randomly without replacement. The probability that the two balls are of different colours is? (Q6, p752; UGC NET Jun 2019)

There are two scenarios here:

Scenario 1: You first find a white ball and then black

Scenario 2: You first find a black ball and then white

In scenario 1, the probability of finding a white ball first is  $\frac{3}{5}$  and the probability of finding a black ball in the second attempt is  $\frac{2}{4}$ .

(4 is in the denominator because, according to the question, we did not replace the white ball after taking it out).

$$\text{Probability of Scenario 1} = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$

In scenario 2, the probability of finding a black ball first is  $\frac{2}{5}$  and the probability of finding a black ball in the second attempt is  $\frac{3}{4}$ . Again, 4 is in the denominator in the latter because we did not replace the black ball after pulling it out.

$$\text{Probability of Scenario 2} = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$$

So, Probability that two balls are of different colours

$$= \text{probability of Scenario 1} + \text{Probability of Scenario 2} = \frac{6}{10} = \frac{3}{5} \text{ or } 0.6$$

112. In a population of 210 individuals, 72 are smoker and 138 are non-smokers. If a person is selected with an equal chance to each category, what is the probability that the person is a smoker? (Q23, p754; UGC NET Nov 2017)

$$\text{Probability of someone being a smoker is } 72 \div 210 = 0.34$$

113. In a health survey, 38 of 120 men and 24 of 80 women are found to be affected by smoking. What is the probability of a randomly selected male being a smoker? (Q54, p757; UGC NET Jul 2016)

If the question says we are randomly selected a male, it gives the impression that we are only targeting the pool of men and randomly selecting within that, and then calculating the probability of selecting a smoker. The question is actually asking the probability of randomly selecting a male smoker from the entire set of 200 people (i.e 120 men + 80 women).

$$\text{So, } 38 \div 200 = 0.19$$

## 21 Statistics (unit 9, p752-766)

### 21.1 Harmonic mean

– 46, 145 [https://www.mathwords.com/h/harmonic\\_mean.htm](https://www.mathwords.com/h/harmonic_mean.htm)

114. What is the harmonic mean of 5, 10 and 20 (UGC NET Aug 2016)

$$\text{Harmonic mean} = \frac{n}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}} = \frac{3}{\frac{1}{5} + \frac{1}{10} + \frac{1}{20}} = \frac{3}{\frac{7}{20}} = 8.57$$

### 21.2 Geometric mean

– 48, 75, 100, 109, 123, 142, 152 <https://www.mathsisfun.com/numbers/geometric-mean.html>

115. What is the geometric mean of 2, 4, 8, 16, 32 (UGC NET Jun 2007 and Q152 in Singh)

Geometric mean of two numbers is  $\sqrt{\text{first number} \times \text{second number}}$

Geometric mean of three numbers is =

$$\sqrt[3]{\text{first number} \times \text{second number} \times \text{third number}}$$

So geometric mean of five numbers is

$$\sqrt[5]{\text{first} \times \text{second} \times \text{third} \times \text{fourth} \times \text{fifth number}}$$

$$= \sqrt[5]{2 \times 4 \times 8 \times 16 \times 32} = \sqrt[5]{2 \times 2^2 \times 2^3 \times 2^4 \times 2^5} = \sqrt[5]{2^{15}} = \sqrt[5]{(2^3)^5} = 2^3 = 8$$

### 21.3 Percentiles, Percentile ranks, Medians, Quartiles

– 155 20, 27 <https://www.youtube.com/watch?v=AdZ8svD7e7s> and Rumsey (2019: 19).

## Finding a percentile

To calculate the  $k$ th percentile (where  $k$  is any number between one and one hundred), do the following steps:

- 1.** Order all the numbers in the data set from smallest to largest.
- 2.** Multiply  $k$  percent times the total number of numbers,  $n$ .
- 3a.** If your result from Step 2 is a whole number, go to Step 4. If the result from Step 2 is not a whole number, round it up to the nearest whole number and go to Step 3b.
- 3b.** Count the numbers in your data set from left to right (from the smallest to the largest number) until you reach the value from Step 3a. This corresponding number in your data set is the  $k$ th percentile.
- 4.** Count the numbers in your data set from left to right until you reach that whole number. The  $k$ th percentile is the average of that corresponding number in your data set and the next number in your data set.

## The Five-Number Summary

The five-number summary is a set of five descriptive statistics that divide the data set into four equal sections. The five numbers in a five number summary are:

- 1.** The minimum (smallest) number in the data set
- 2.** The 25th percentile, aka the first quartile, or Q1
- 3.** The median (or 50th percentile)
- 4.** The 75th percentile, aka the third quartile, or Q3
- 5.** The maximum (largest) number in the data set

21.4 Standard error of the mean aka margin of error

– 51, 55

116. For a sample size (N) of 64, the mean and standard deviation were 82 and 6. The standard error is? (UGC Jul 2016)

Standard error of the mean = standard deviation ÷ sqrt(n)

$$= \frac{6}{\sqrt{64}} = 6/8 = 0.75$$

21.5 Standard deviation

– 43

The formula for the standard deviation of a data set is

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}}$$

To calculate s, do the following steps:

1. **Find the average of the data set,  $\bar{x}$ .** To find the average, add up all the numbers and divide by the number of numbers in the data set,  $n$ .
2. **For each number, subtract the average from it.**
3. **Square each of the differences.**
4. **Add up all the results from Step 3.**
5. **Divide the sum of squares (Step 4) by the number of numbers in the data set, minus one ( $n - 1$ ).**

If you do Steps 1 through 5 only, you have found another measure of variability, called the *variance*.

6. **Take the square root of the variance. This is the standard deviation.**

Suppose you have four numbers: 1, 3, 5, and 7. The mean is  $16 \div 4 = 4$ . Subtracting the mean from each number, you get  $(1 - 4) = -3$ ,  $(3 - 4) = -1$ ,  $(5 - 4) = +1$ , and  $(7 - 4) = +3$ .

Squaring the results you get 9, 1, 1, and 9, which sum to 20.

Divide 20 by  $4 - 1 = 3$  to get 6.67. The standard deviation is the square root of 6.67, which is 2.58.

117. In a city, the daily per capita inhalation ( $m^3$ ) values of contaminated air over a period of five days are 6.3, 6.3, 6.2, 6.4 and 5.8. The sample mean with standard deviation of the data is. (Q43, p755)

Inhalations per day	$X - \bar{x}$	$(X - \bar{x})^2$
6.3	0.1	0.01
6.3	0.1	0.01
6.2	0	0
6.4	0.2	0.04
5.8	-0.4	0.16
$\bar{x} = 6.2$	<b>0</b>	<b>0.22</b>

## Numerical Questions

Note that each  $(X-\bar{x})$  has to be squared individually and then totalled. You cannot add all  $(X-\bar{x})$  values and then square them up.

$$\text{Standard deviation (s)} = \sqrt{\frac{\sum(X-\bar{x})^2}{n-1}} = \sqrt{\frac{0.22}{4}} = 0.23$$

### 21.6 combined standard deviation and/or variance

– 12, 81

If  $x, y$  are arithmetic means of two datasets containing  $p$  and  $q$  items respectively and  $a$  and  $b$  are standard deviations, and  $s$  is the standard deviation of the combined data, then

$$\text{standard deviation (s)} = \sqrt{\frac{pa^2+qb^2}{p+q} + \frac{pq(x-y)^2}{(p+q)^2}}$$

$$\text{variance (s}^2\text{)} = \frac{pa^2 + qb^2}{p + q} + \frac{pq(x-y)^2}{(p + q)^2}$$

118. In each experiment, five measurements were made of the observable. The first experiment yielded mean and standard deviation 80 and 4 respectively, while the second experiment gave mean and standard deviation 100 and 5 respectively. What is the combined standard deviation of the data? (Q12, p753)

$$\begin{aligned} s &= \sqrt{\frac{pa^2 + qb^2}{p + q} + \frac{pq(x-y)^2}{(p + q)^2}} \\ &= \sqrt{\frac{5 \times 4^2 + 5 \times 5^2}{5+5} + \frac{(5 \times 5)(80-100)^2}{(5+5)^2}} = \sqrt{120.5} = 10.977 \end{aligned}$$

Also see <https://www.youtube.com/watch?v=PE9yiCbSNF8> Q8

### 21.7 Confidence interval

– 9, 15, 24, 63, 71, 106, 112, 114

When you know the population standard deviation and mean, confidence interval is

$\bar{x} \pm z \left( \frac{\sigma}{\sqrt{n}} \right)$  or  $\bar{x} \pm t \left( \frac{\sigma}{\sqrt{n}} \right)$  where  $\bar{x}$  is sample mean,  $\sigma$  is population standard deviation and  $n$  is sample size.

Confidence Level	$z^*$ -value
80%	1.28



90%	1.645 (by convention)
95%	1.96
98%	2.33
99%	2.58

119. Based on random assessment of four years, the mean net production and standard deviation for a grassland are 120 gC/m<sup>3</sup> per year and 4 gC/m<sup>3</sup> per year respectively. The 90% confidence interval around this estimate is (t at 10% level of significance for df 3 is 2.353)? (Q9, p752)

$$\bar{x} \pm t \left( \frac{\sigma}{\sqrt{n}} \right) = 120 \pm 2.353 \left( \frac{4}{\sqrt{4}} \right) = 120 \pm 4.706$$

**Size of sample** – 60, 117

## 21.8 Regression

– 3, 17, 29, 40, 124, 140, 143, 156

In the case of two numerical variables, it's possible to come up with a line that you can use to predict Y from X, if (and only if) the following two conditions you examined in the previous sections are met: 1) The scatterplot must find a linear pattern; and 2) The correlation, r, is moderate to strong (typically beyond 0.60).

The formula for the best-fitting line (or regression line) is  $y = mx + b$ , where m is the slope of the line and b is the y-intercept (Rumsey 2019: 119-120).

## Finding the slope

The formula for the slope, m, of the best-fitting line is  $m = r \left( \frac{s_y}{s_x} \right)$

where r is the correlation between X and Y, and  $s_x$  and  $s_y$  are the standard deviations of the x-values and the y-values. To calculate the slope, m, of the best-fitting line:

1. Divide  $s_y$  by  $s_x$ .
2. Multiply the result in Step 1 by r.

The correlation and the slope of the best-fitting line are not the same. The formula for slope takes the correlation (a unit-less measurement) and attaches units to it. Think of  $s_y / s_x$  as the change in Y over the change in X, in units of X and Y; for example, change in temperature (degrees Fahrenheit) per increase of one cricket chirp (in 15 seconds).

## Finding the y-intercept

The formula for the  $y$ -intercept,  $b$ , of the best-fitting line is  $b = \bar{y} - m\bar{x}$ , where  $\bar{x}$  and  $\bar{y}$  are the means of the  $x$ -values and the  $y$ -values, respectively, and  $m$  is the slope (the formula for which is given in the preceding section). To calculate the  $y$ -intercept,  $b$ , of the best-fitting line:

- 1. Find the slope,  $m$ , of the best-fitting line using the steps listed in the preceding section.**
- 2. Multiply by  $\bar{x}$ .**
- 3. Subtract your result from  $\bar{y}$ .**

### 21.9 Correlation coefficient

– Qs 1, 47, 60, 64, 94, 151, 153, 156

[https://www.youtube.com/watch?time\\_continue=565&v=2SCg8Kuh0tE&feature=emb\\_title](https://www.youtube.com/watch?time_continue=565&v=2SCg8Kuh0tE&feature=emb_title)

The formula for the correlation ( $r$ ) is

$$r = \frac{1}{n-1} \sum \frac{(x - \bar{x})(y - \bar{y})}{s_x s_y}$$

where  $n$  is the number of pairs of data;  $\bar{x}$  and  $\bar{y}$  are the sample means; and  $s_x$  and  $s_y$  are the sample standard deviations of the  $x$ - and  $y$ - values, respectively.

To calculate the correlation  $r$  from a data set:

- 1. Find the mean of all the  $x$ -values ( $\bar{x}$ ) and the mean of all the  $y$ -values ( $\bar{y}$ ).**  
See Chapter 2 for information on the mean.
- 2. Find the standard deviation of all the  $x$ -values (call it  $s_x$ ) and the standard deviation of all the  $y$ -values (call it  $s_y$ ).**  
See Chapter 2 for information on standard deviation.
- 3. For each  $(x, y)$  pair in the data set, take  $x$  minus  $\bar{x}$  and  $y$  minus  $\bar{y}$ , and multiply them together.**
- 4. Add up all the results from Step 3.**
- 5. Divide the sum by  $s_x * s_y$ .**
- 6. Divide the result by  $n - 1$ , where  $n$  is the number of  $(x, y)$  pairs.**

This gives you the correlation  $r$ .

120. Simultaneous measurements of humidity Y (%) and temperature X (%) at a given site are given as under. What is the correlation coefficient? (Q1, p752)

<b>Humidity (Y)</b>	30	40	45	60	65	
<b>Temperature (X)</b>	40	35	30	25	20	
<b>Average Humidity (<math>\bar{y}</math>)</b>	48	48	48	48	48	
<b>Average Temperature (<math>\bar{x}</math>)</b>	30	30	30	30	30	
<b>Y-<math>\bar{y}</math></b>	-18	-8	-3	12	17	
<b>X-<math>\bar{x}</math></b>	10	5	0	-5	-10	
<b>(Y-<math>\bar{y}</math>)<sup>2</sup></b>	324	64	9	144	289	<b><math>\Sigma(Y-\bar{y})^2=830</math></b>
<b>(X-<math>\bar{x}</math>)<sup>2</sup></b>	100	25	0	25	100	<b><math>\Sigma(X-\bar{x})^2 = 250</math></b>
<b>(Y-<math>\bar{y}</math>)(X-<math>\bar{x}</math>)</b>	-180	-40	0	-60	-170	<b><math>\Sigma(Y-\bar{y})(X-\bar{x}) = -450</math></b>

$$\text{Standard deviation for humidity} = S_y = \sqrt{\frac{\Sigma(Y-\bar{y})^2}{n-1}} = \sqrt{\frac{830}{5-1}} = \sqrt{\frac{830}{4}} = 14.40$$

$$\text{Standard deviation for temperature} = S_x = \sqrt{\frac{\Sigma(X-\bar{x})^2}{n-1}} = \sqrt{\frac{250}{5-1}} = \sqrt{\frac{250}{4}} = 7.90$$

$$S_y S_x = 113.76$$

$$\text{Correlation coefficient, } r = \left[ \frac{\Sigma(Y-\bar{y})(X-\bar{x})}{(n-1)(S_y S_x)} \right] = \frac{-450}{(4 \times 113.76)} = -450 \div 455.04 = -0.988$$

### 21.10 Residual (unexplained) and overall variance

– Qs 3, 8, 91, 101\* (\* = covariance), <https://www.statisticshowto.com/residual-variance/>

Unexplained + Explained variance = 1000

Standard error =  $\sqrt{\text{unexplained variance} \div \text{number of observations}}$

121. In a simple regression analysis of y on x, the standard error of the estimate of y on x,  $s_{yx} = 5$ , number of observations (N) is 30 and  $\Sigma y^2 = 2000$ . The unexplained variance is: (Q91, p760; <https://www.youtube.com/watch?v=o98z2aopJql>)

$$\text{Standard error} = \sqrt{\frac{\text{unexplained variance}}{\text{number of observations}}}$$

$$5 = \sqrt{\text{unexplained variance} \div 30}$$

Squaring both sides, we get  $25 = \text{unexplained variance} \div 30$

$$\text{Unexplained variance} = 750$$

**Matrices** – 84, 86, 89, 96, 125, 130

21.11 Chi sq or a goodness of fit test

– 16, 62, 66, 97, 115

When data is given,  $\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$

When sample standard deviation (s) and population standard deviation ( $\sigma$ ) are given,

$$\text{if } n < 30, \text{ then } \chi^2 = \frac{n \times s^2}{\sigma^2} \text{ and if } n > 30, \text{ then } \chi^2 = \frac{(n - 1) \times s^2}{\sigma^2}$$

When sum of squares of deviation is given, then

$$\chi^2 = \frac{\text{sum of squares of deviations}}{\text{variance}} = \frac{\sum(x - \bar{x})^2}{\sigma^2}$$

122. In a fish population in a pond it is believed that males and females are in equal proportion. If out of 200 fish in a catch, 120 are male and 80 are female, what is the  $\chi^2$  statistic? (Q16, p753) (e.g. <https://www.youtube.com/watch?v=V4SRgabFbz0>)

	Male	Female	
Observed	120	80	
Expected	100	100	
Observed – Expected	20	-20	
(Observed – Expected) <sup>2</sup>	400	400	
{(Observed – Expected) <sup>2</sup> ÷ Expected}	400/100 = 4	400/100 = 4	$\sum\{(\text{Observed} - \text{Expected})^2 / \text{Expected}\} = 8$

123. A normal population  $\sigma^2 = 6$ . The sum of squares of deviations of 15 sample values from their mean being 120, what is the  $\chi^2$  value? (Q66, p758) ([https://www.youtube.com/watch?v=cPd\\_yVW5YbA](https://www.youtube.com/watch?v=cPd_yVW5YbA))

$$\chi^2 = \frac{\text{sum of squares of deviations}}{\text{variance}} = \frac{120}{6} = 20$$

124. The standard deviation of weights of certain 1 kg packets of milk is 10 grams. A random sample of 20 packets showed a standard deviation of 15 grams. The value of  $\chi^2$  statistic for the sample is? (Q97, p761) (cf. <https://www.youtube.com/watch?v=q1BYFLqYYiI>)

$$\chi^2 = \frac{n \times s^2}{\sigma^2} = \frac{20 \times 15^2}{10^2} = 45$$

**p-value** – 25 (Rumsey 2019: 92-94)

21.12 ANOVA and f-statistic

– 22, 52, 65, 92, 103, 111, 118, 150 <https://www.youtube.com/watch?v=srDr-4cz1KI>  
<https://www.statisticshowto.com/probability-and-statistics/f-statistic-value-test/>  
<https://www.statisticshowto.com/probability-and-statistics/hypothesis-testing/anova/>

## Numerical Questions

125. In a one-way ANOVA, explained variance was 8.0 and unexplained variance was 3.67. The f-ratio is? (UGC NET Nov 2017, p754)

$$f\text{-ratio} = \text{explained variance} \div \text{unexplained variance} = 8 \div 3.67 = 2.17$$

*Tip: in ANOVA, intra group variation and residual variance refer to the same thing.*

126. Two normal populations have variances  $\sigma_1^2 = 10$  and  $\sigma_2^2 = 20$ , two random samples of sizes 25 and 20, independently selected from these populations have variances of  $S_1^2 = 8$  and  $S_2^2 = 15$  respectively. Calculate  $F_{(24, 19)}$  ratio (UGC NET Dec 2013, p761)

$$F \text{ ratio} = \{(s_1^2 \div \sigma_1^2) \div (s_2^2 \div \sigma_2^2)\} = \{(8 \div 10) \div (15 \div 20)\} = 0.8 \div 0.75 = 1.06$$

127. In a multiple regression model, explained variance per degree of freedom is 50 and unexplained (aka residual) variance per degree of freedom is 10. F ratio is? (UGC NET Dec 2012, p763; Q103, p761 can also be solved the same way).

$$f\text{-ratio} = \text{explained variance} \div \text{unexplained variance} = 5$$

### 21.13 t-statistic

-4, 10, 45, 80\*, 82, 88\*, 98 [https://www.youtube.com/watch?v=uJ2Yy5gA\\_oc](https://www.youtube.com/watch?v=uJ2Yy5gA_oc), 128\*, 161\* (\* = t distribution)

$$t \text{ statistic} = [ \{ (\text{sample mean} - \text{population mean}) \times \sqrt{n} \} \div \text{standard deviation} ]$$

128. For a sample size  $n = 16$ , the mean and standard deviations are 5.667 and 0.934 respectively. If the population mean (aka true mean) is 5.2, then t statistic is? (Q45, p756; UGC NET JAN 2017)

$$\begin{aligned} \text{Using } t \text{ statistic} &= [ \{ (\text{sample mean} - \text{population mean}) \times \sqrt{n} \} \div \text{standard deviation} ], \\ &= [ \{ (5.667 - 5.2) \times \sqrt{16} \} \div 0.934 ] = [ \{ (0.467 \times 4) \} \div 0.934 ] = 2.0 \end{aligned}$$

129. A random sample of size 26 has a mean of 20. The sum of squares of the deviation taken from the mean is 200. If the population mean is 18, what is the value of t? ([https://www.youtube.com/watch?v=uJ2Yy5gA\\_oc](https://www.youtube.com/watch?v=uJ2Yy5gA_oc)).

$$\text{Standard deviation} = \sqrt{\text{sum of squares of deviation} \div (\text{number of observations} - 1)}$$

$$\text{Standard deviation} = \sqrt{200 \div 25} = \sqrt{8}$$

$$\begin{aligned} \text{Using } t \text{ statistic} &= [ \{ (\text{sample mean} - \text{population mean}) \times \sqrt{n} \} \div \text{standard deviation} ], \\ &= [ \{ (20 - 18) \times \sqrt{26} \} \div \sqrt{8} ] = 3.61 \end{aligned}$$

21.14 significance level

– 50, 58, 60

130. For sampling error of  $1.96\sigma$ , where  $\sigma$  is the standard deviation, and at critical value 1.96, the significance level is? (Q58, p757) cf.

<https://www.youtube.com/watch?v=Tx74YQvkPow>.

Confidence level	$\alpha$ (level of significance)	Z score aka critical value
99%	1%	2.575
95%	5%	1.96
90%	10%	1.645

Since the critical value is 1.96, the corresponding level of significance is 5%.

**degrees of freedom** – 7, 115

**poisson distribution** – 7, 69, 107, 110

**binomial distribution, probability of getting  $x$  successes in  $n$  trials**

Probabilities for a binomial random variable  $X$  can be found using

the formula  $\binom{n}{x} p^x (1-p)^{n-x}$ , where

- »  $n$  is the fixed number of trials.
- »  $x$  is the specified number of successes.
- »  $n - x$  is the number of failures.
- »  $p$  is the probability of success on any given trial.
- »  $1 - p$  is the probability of failure on any given trial. (**Note:** Some textbooks use the letter  $q$  to denote the probability of failure rather than  $1 - p$ .)

The number of ways to arrange  $x$  successes among  $n$  trials is called “ $n$  choose  $x$ ,” and the notation is  $\binom{n}{x}$ . For example,  $\binom{3}{2}$  means “3 choose 2” and stands for the number of ways to get 2 successes in 3 trials. In general, to calculate “ $n$  choose  $x$ ,” you use the formula

$\binom{n}{x} = \frac{n!}{x!(n-x)!}$ . The notation  $n!$  stands for  $n$ -factorial, the number of ways to rearrange  $n$  items. To calculate  $n!$ , you multiply  $n(n-1)(n-2)\dots(2)(1)$ . For example  $3!$  is  $3(2)(1) = 6$ ;  $2!$  is  $2(1) = 2$ ; and  $1!$  is 1. By convention,  $0!$  equals 1. To calculate “3 choose 2,” you do the following:

$$\binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3 \cdot 2 \cdot 1}{(2 \cdot 1)(1!)} = \frac{6}{2 \cdot 1} = 3$$

#### 21.15 binomial distribution, mean and SD or variance

– Q157, p766

The mean and variance of a binomial have intuitive meaning. The  $p$  is the probability of a success, but it also represents the proportion of successes you can expect in  $n$  trials. Therefore, the total number of successes you can expect — that is, the mean of  $X$  — equals  $np$ . The only variability in the outcomes of each trial is between success (with probability  $p$ ) and failure (with probability  $1-p$ ). Over  $n$  trials, it makes sense that the variance of the number of successes/failures is measured by  $np(1-p)$  (Rumsey 2019: 43).

*mean = number of trials ( $n$ )  $\times$  probability of success ( $p$ )*

Suppose you flip a fair coin 100 times and let  $X$  be the number of heads; this is a binomial random variable with  $n$  100 and  $p$  0.50. Its mean is  $np$  100(0.50) 50.

Standard deviation ( $s$ ) =  $\sqrt{np(1-p)}$

Variance ( $s^2$ ) =  $\{np(1-p)\}$

#### **Finding z-value for a normal distribution**

Z value = (value of  $x$  – mean)/standard deviation

## Numerical Questions

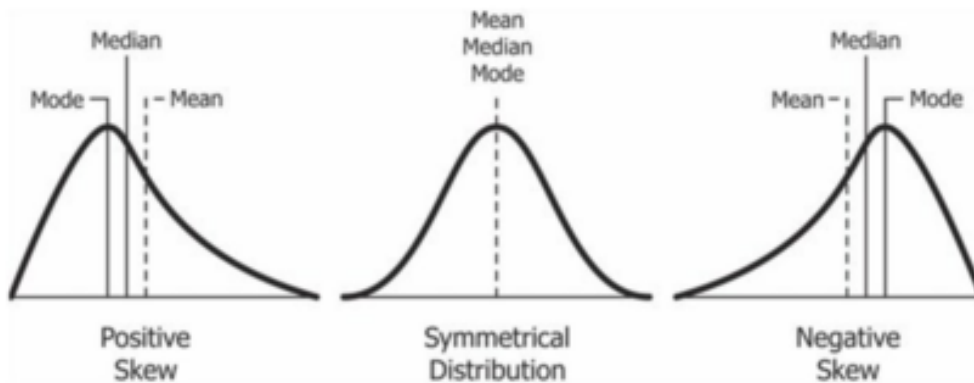
### 21.16 Skewness

– Q147, 26, 28, 5

If the distribution is skewed, then  $\text{mean} \neq \text{median} \neq \text{mode}$

Positively skewed:  $\text{mean} > \text{median} > \text{mode}$

Negatively skewed:  $\text{mean} < \text{median} < \text{mode}$



### 21.17 Moments of distribution

– Q102, p761; Q144, p765;

The shape of any distribution can be described by its various ‘moments’. The first four are:

- 1) The mean, which indicates the central tendency of a distribution.
- 2) The second moment is the variance, which indicates the width or deviation.
- 3) The third moment is the skewness, which indicates any asymmetric ‘leaning’ to either left or right.
- 4) The fourth moment is the Kurtosis, which indicates the degree of central ‘peakedness’ or, equivalently, the ‘fatness’ of the outer tails. (<https://www.risk.net/definition/moments-statistical-distribution>)

Singh (2019: 419)

Dispersion – 158

Z transformation – 160